Introduction to Scientific Computing, PSCB57, Fall 2018
Assignment 5
Differential Equations

Instructions

- You must submit the assignment electronically via Quercus. The deadline for this assignment is Monday, November 19th, 9am. Late assignments will not be accepted unless accompanied by supportive documentation.
- This assignment comes in multiple parts. Submit all your answers in one Jupyter Notebook file with the file type `ipynb`.
- Use mark-down cells to add your name, student number. Also use mark-down cells and python comments to describe your code! Well documented code might help you with the quiz.
- Do not use any packages or libraries other than numpy and matplotlib in this assignment.
- You need to implement the Euler Method and the Runge Kutta method yourself (do not use a package for that).
- You must be present at the tutorial on Tuesday where you will be quizzed about your assignment. If you do not show up or fail to pass the quiz, your assignment might be marked as 0% even if it was correct.
- Plagiarism is taken very seriously. However, you are not expected to work in solitude and are encouraged to talk to your classmates. But keep in mind that if you submit an assignment, you have to fully understand it in order to pass the quiz.

Part 1

Write a function to integrate the following function

$$\dot{x} = x^2 - 4x + 5$$

with the following initial condition $x(-1) = 0$ by applying the Euler Method to the interval $[-1, 1]$ with n time steps.

Vary n and explore how the solution and runtime change. Use this to estimate the error and choose a value for n such that the *relative error* is approximately 1% at the end of the integration. Describe how you estimate the error.

Part 2

Similarly write a function to implement the Runge-Kutta stepping method to solve the same differential equation using the same number of time steps. How do the runtime and the relative error change?

Part 3 (optional)

An apartment building holds 500 people. After receiving 2 metres of snow, everyone is trapped in the building. Some of the apartment's dwellers suddenly become zombies. The zombies go rampaging through the building, turning the other apartment dwellers into zombies.

Fortunately, nine people know how to kill zombies, and they are able to teach the other people who live in the apartment building. The chance of surviving a zombie encounter differs between zombie killers and non-zombie killers, but if a person doesn't survive the encounter, they become a zombie. You can model this using the following equations:

$$\begin{aligned}
\frac{\partial S}{\partial t} &= -b\, S(t)Z(t) - e\, S(t)K(t) \\
\frac{\partial K}{\partial t} &= -c\, K(t)Z(t) + e\, S(t)K(t) \\
\frac{\partial Z}{\partial t} &= b\, S(t)Z(t) + c\, K(t)Z(t) - a\, K(t)Z(t)
\end{aligned}$$

where S is the number of regular people who can't kill zombies; K is the number of zombie killers; Z is the number of zombies; a is the rate at which zombies are killed by K; b is the rate at which regular people are turned into zombies; c is the rate at which zombie killers are turned into zombies; and e is the rate at which zombie killers teach regular people how to kill zombies. The following rates are all given using a time unit of 1 day:

$$a = 0.03, \quad b = 0.02, \quad c = 0.01, \quad \text{and } e = 0.015.$$

Use one of your functions from Part 1 or 2 to test out different starting zombie populations and solve the system of equations up to t = 10 days. What is the minimum number of initial zombies, Z_0, that it takes to turn the whole building population into zombies? What is the maximum number of initial zombies that will allow the zombies to be eliminated?

Produce populations versus time plots for the following scenarios: (1) zombies win and all humans disappear; (2) humans win and zombies disappear; and (3) humans and zombies live in harmony.

Note that the total number of people in the building is 500, there are 9 zombie killers initially, so that $S(0) = 491 - Z(0)$.