

Name: _____

Student number: _____

Introduction to Scientific Computing PSCB57, Final Exam

Professor Hanno Rein
University of Toronto, Scarborough
Saturday, 19th December 2015, SW143, 7pm-9pm

	Points	Max Points
Question 1		6
Question 2		4
Question 3		3
Question 4		2
Question 5		3
Question 6		3
Question 7		3
Question 8		6
Question 9		4
Question 10		2
Question 11		5
Question 12		3
Question 13		4
Question 14		4
Total		52

- No calculators, aid sheets, books or other notes are allowed.
- All electronic devices must be stored together with your belongings at the back of the room.
- Write your answers on the question sheet. If you need more paper raise your hand.
- The length of the white space for each question gives you an idea of the expected length and complexity of the answer.
- If you leave the room, please do not disturb others. You can not leave in the last 10 minutes.
- The University of Toronto's Code of Behaviour on Academic Matters applies to all University of Toronto Scarborough students. The Code prohibits all forms of academic dishonesty including, but not limited to, cheating, plagiarism, and the use of unauthorized aids. Students violating the Code may be subject to penalties up to and including suspension or expulsion from the University.

Question 1

6 Points

The combined radioactivity $R(t)$ of a radioactive substance and a constant amount of background radiation is measured in events per second and can be described with the functional form

$$R(t) = R_0 \cdot e^{-t/\tau_A} + R_C.$$

For the substance we are considering here the half-life constant τ_A is 10 yrs. The decay rate $R(t)$ has been measured at three times:

Time [yrs]	Decay rate [1/s]
0	111
12.039	32
16.094	37

Use a linear least square fit to find the best fit values for R_0 and R_C . Note that you can solve the linear system of equations for the least square fit without the LU decomposition in this case. You may assume that $e^{-1.2039} \approx 0.3$ and $e^{-1.6094} \approx 0.2$.

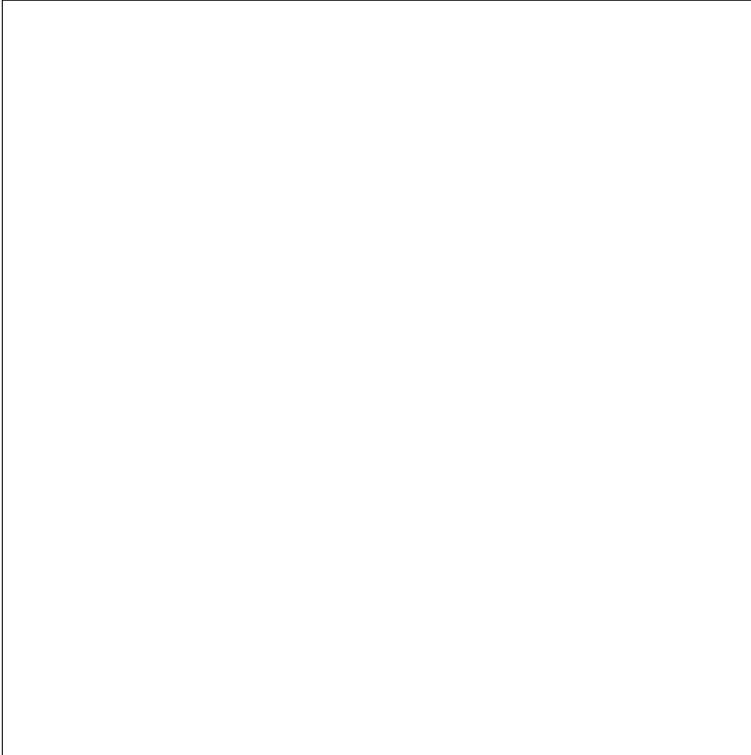
$R_0 =$ _____

$R_C =$ _____

Question 2

4 Points

The height of 40 individuals has been measured. The dataset is given below. Draw a diagram to visualize this dataset. It is not necessary to draw a diagram that is quantitatively correct. Only the conceptual and stylistic choices you make matter.



Dataset:

65.78 in	67.83 in	66.49 in	69.52 in
71.52 in	68.88 in	67.62 in	65.81 in
69.40 in	63.48 in	68.30 in	67.82 in
68.22 in	68.42 in	67.12 in	70.60 in
67.79 in	67.63 in	68.28 in	71.80 in
68.70 in	67.21 in	71.09 in	69.21 in
69.80 in	70.84 in	66.46 in	66.80 in
70.01 in	67.49 in	68.65 in	67.66 in
67.90 in	66.53 in	71.23 in	67.81 in
66.78 in	65.44 in	67.13 in	64.05 in

Question 3

3 Points

You are given a dataset and the two functions

$$f(x) = a_0 \sin(a_1 \cdot x + a_2)$$

$$g(x) = a_0 [Y_4(x)]^{5.3} + a_1 \cdot \log(\sqrt{x}), \text{ where } Y_4 \text{ is a Bessel function of second kind.}$$

Can you use a linear least square fit to fit these functions to the dataset? If not, can you rewrite them so that you can use a standard linear least square fit?

Question 4

2 Points

What is the difference between a fit and an interpolation?

Question 5

3 Points

Find the Lagrange polynomial that interpolates the following (x, y) points. Do not simplify the polynomial.

$$\begin{aligned}(0, 3) \\ (\sqrt{2}, 4^{-1/3}) \\ (6, 7) \\ (-8, \pi)\end{aligned}$$

What would be the order of the polynomial if you were given 2 datapoints instead of 4? _____

Question 6

3 Points

Describe a scenario where you would use a cubic spline to interpolate a dataset. What properties should the dataset have?

What are the advantages of a cubic spline compared to

1) linear interpolation?

2) Lagrange interpolation?

Question 7

3 Points

```
def f(x):  
    return x*x-2.  
def df(x):  
    return 2.*x  
x = 3.  
for i in xrange(10):  
    x -= f(x)/df(x)  
print x
```

State the mathematical problem that the above python code solves:

What is the name of the method? _____

Name at least one other method we discussed in the lectures that can solve this kind of problem:

Question 8

6 Points

Rewrite the following second order differential equation as a first order differential equation:

$$f''(x) = -(2\pi)^2 f(x).$$

Use the initial conditions $f(0) = 0$, $f'(0) = 1$ and a timestep of $dx = 0.25$ to integrate the system for two timesteps using the Euler method.

What is your result for $f(0.5)$? _____

What is the exact solution to the differential equation evaluated at $x = 0.5$? _____

Describe two ways how you could improve the numerical result:

1) _____

2) _____

Question 9

4 Points

We work in IEEE754 double floating point precision. Circle all numbers that can be represented exactly.

1.1 1024.0 0.0 $-1e-1$ $-1e+1$

0.125 0.1024 $1e+1024$ 0.5 10.25

Calculate the following expressions.

$(1e+200 / 1e-187) + 1.5 =$ _____

$((1e+18 + 1.125) - 1e18) + 1e-18 =$ _____

Question 10

2 Points

```
SET 1 r8
PRINT r8
SET 1 r5
PRINT r5
ADD r8 r5 r2
COPY r5 r8
PRINT r2
COPY r2 r5
SET -5 r3
JUMP r3
```

What is the above assembler program doing? (The syntax of assembler commands can be found in the appendix.)

Will the program terminate? If so, what is the last number printed before it exits? If not, speculate what will happen.

Question 12

3 Points

Next to every YouTube video is a counter that shows how many people have watched the video. On December 1st 2014, the counter for the video *Psy - Gangnam Style* stopped working after 2 147 483 647 people watched it. Explain what happened.

Speculate which number was shown to the next person watching the video?

What could Google do to prevent this bug in the future?

Question 13

4 Points

```
# Main loop
for i in xrange(5000):

    theta_proposed = [theta+0.1*numpy.random.randn()
                      for theta in theta_current]
    P_proposed = P(Dx,Dy,theta_proposed)

    # Calculate likelihood ratio (uses log likelihood)
    ratio = math.exp(P_proposed - P_current)

    # Accept the new theta values if likelihood is larger
    if ratio>1.:
        theta_current = theta_proposed
        P_current = P_proposed

    # Save current theta value in chain
    chain.append(theta_current[0])

# End main loop
```

The above python code shows the main loop of an implementation of the Metropolis-Hastings algorithm. The code is missing an important part of the algorithm. Propose a fix (you can write it directly into the code or below). Explain in which case the algorithm would not work without your fix.

Question 14

4 Points

In the MCMC code from the lectures, we use the log likelihood of the form

$$\mathcal{L} \equiv - \sum_i (y_i - m(x_i))^2$$

instead of the actual likelihood function

$$p \equiv \exp \left(- \sum (y_i - m(x_i))^2 \right).$$

We can use both forms to calculate the probability ratio needed to decide on the acceptance of the proposal. When using the logarithmic form we have

$$r = \exp(\mathcal{L}_p - \mathcal{L}_0)$$

otherwise

$$r = p_p/p_0.$$

Mathematically, these expressions are equivalent. Use a concrete example (including data and model) to explain why it is often better to work with the log likelihood on a computer.

Appendix A Assembler commands

- ADD r1 r2 r3
This command adds the value stored in register r1 to the value stored in r2 and stores the result in r3.
- IF r1 r2
This command only executes the next command if the value in register r1 is bigger than the value in register r2.
- COPY r1 r2
This command copies the value in register r1 to register r2.
- SET s1 r1
This command sets the value in register r1 to the number s1 (s1 is just a number, not the address of a register).
- JUMP r1
This command moves the instruction pointer by the value of r1.
- PRINT r1
This command prints the value in register r1 on the screen.

Appendix B Powers of two

$2^0 = 1$	$2^{21} = 2097152$	$2^{42} = 4398046511104$
$2^1 = 2$	$2^{22} = 4194304$	$2^{43} = 8796093022208$
$2^2 = 4$	$2^{23} = 8388608$	$2^{44} = 17592186044416$
$2^3 = 8$	$2^{24} = 16777216$	$2^{45} = 35184372088832$
$2^4 = 16$	$2^{25} = 33554432$	$2^{46} = 70368744177664$
$2^5 = 32$	$2^{26} = 67108864$	$2^{47} = 140737488355328$
$2^6 = 64$	$2^{27} = 134217728$	$2^{48} = 281474976710656$
$2^7 = 128$	$2^{28} = 268435456$	$2^{49} = 562949953421312$
$2^8 = 256$	$2^{29} = 536870912$	$2^{50} = 1125899906842624$
$2^9 = 512$	$2^{30} = 1073741824$	$2^{51} = 2251799813685248$
$2^{10} = 1024$	$2^{31} = 2147483648$	$2^{52} = 4503599627370496$
$2^{11} = 2048$	$2^{32} = 4294967296$	$2^{53} = 9007199254740992$
$2^{12} = 4096$	$2^{33} = 8589934592$	$2^{54} = 18014398509481984$
$2^{13} = 8192$	$2^{34} = 17179869184$	$2^{55} = 36028797018963968$
$2^{14} = 16384$	$2^{35} = 34359738368$	$2^{56} = 72057594037927936$
$2^{15} = 32768$	$2^{36} = 68719476736$	$2^{57} = 144115188075855872$
$2^{16} = 65536$	$2^{37} = 137438953472$	$2^{58} = 288230376151711744$
$2^{17} = 131072$	$2^{38} = 274877906944$	$2^{59} = 576460752303423488$
$2^{18} = 262144$	$2^{39} = 549755813888$	$2^{60} = 1152921504606846976$
$2^{19} = 524288$	$2^{40} = 1099511627776$	$2^{61} = 2305843009213693952$
$2^{20} = 1048576$	$2^{41} = 2199023255552$	$2^{62} = 4611686018427387904$

