Name: ______________________________ Student number: ______________

Classical Mechanics
PHYB54
Midterm

Professor Hanno Rein
University of Toronto, Scarborough

Thursday, March 2nd 2017
Duration: 60 minutes

	Points	Max Points
Question 1		4
Question 2		5
Question 3		6
Question 4		8
Total		23

- No aid sheets, books or other notes are allowed.
- All electronic devices must be stored together with your belongings at the back of the room.
- Write your answers on the question sheet. If you need more paper raise your hand.
- The University of Toronto's Code of Behaviour on Academic Matters applies to all University of Toronto Scarborough students. The Code prohibits all forms of academic dishonesty including, but not limited to, cheating, plagiarism, and the use of unauthorized aids. Students violating the Code may be subject to penalties up to and including suspension or expulsion from the University.

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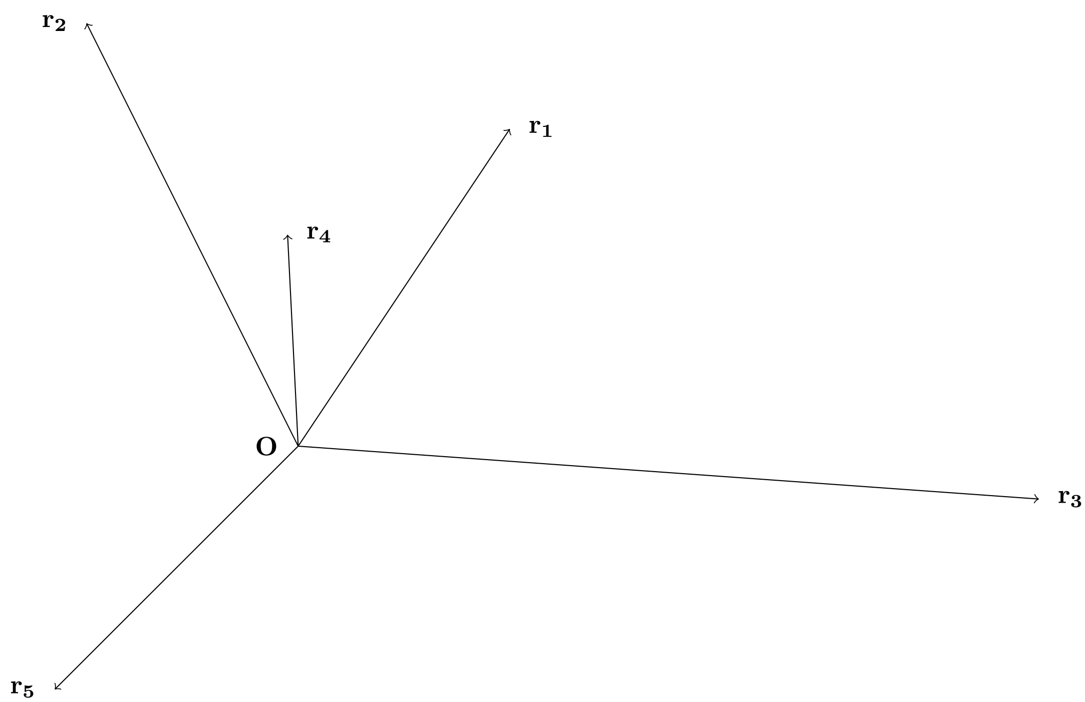
Question 1 **4 Points**

Suppose we have five point particles with the masses $m_1 = m_2 = 1$, $m_3 = 4$, $m_4 = 2$, $m_5 = 8$ and the locations $\mathbf{r}_1, \ldots, \mathbf{r}_5$.

(a) Write down the equation for the centre of mass $\mathbf{R}$ of the five particles. Indicate clearly if a quantity is a vector or not and define all symbols you use that are not given as part of the question.

(b) Assume the locations are given by the vectors shown below. Determine the location of the centre of mass geometrically.



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Question 2 **5 Points**

We consider an N particle system where the particles are pair-wise interacting with each other with some force F. The potential energy between any two particles α and β is $U_{\alpha\beta}(|\mathbf{r}_a - \mathbf{r}_\beta|)$. What can you say about the force F?

Give an example of such a force (name and equation).

What is the total potential energy of the system in terms of $U_{\alpha\beta}$? Be very explicity with the summation indicies.

Question 3 **6 Points**

The four video frames on the left show our rocket launch on the roof of the Science Wing. The frames are 75 milliseconds apart. Use these images and the scale on the left to estimate the acceleration of the rocket. Even though this will obviously be only a rough estimate, try to make it as accurate as possible but don't worry about any perspective distortions.

$a \approx$ ____

The weight of the rocket (including the rocket engine) is 68 g. What is the net force (gravity and thrust) felt by the rocket?

$F_{\text{net}} \approx$ ____

The manufacturer rates the engine with an average thrust of 2.3 N and a maximum thrust of 9.7 N. What is your estimate for the thrust based on the first 0.15 s of the flight? Use $g = 9.8\ \text{m/s}^2$.

$F_{\text{thrust}} \approx$ ____

What height will the rocket reach after 0.7 s? Assume the thrust, the acceleration and mass of the rocket are constant.

$h_{0.7} \approx$ ____

Use the Work-KE theorem to calculate the velocity of the rocket at $t = 0.7$ s.

$v_{0.7} \approx$ ________________________________

The rocket engine turns off after 0.7 s. The manufacturer claims the rocket can reach a height of up to 330 m. Use the Work-KE theorem again to calculate the maximum height above ground (i.e. when the velocity is zero) for our rocket.

$h_{\max} \approx$ ________________________________

(We did not use the most powerful rocket engine available for this type of rocket.)

Question 4 **8 Points**

Write down the Lagrangian for a particle with mass m that can move along the x-axis and is attached to a spring with spring constant k.

$\mathcal{L} =$ ____

Derive the equations of motion using the Lagrange equation.

Write down the general solution to the equations of motion in terms of complex exponentials. What condition(s) do you have to impose on the parameters?

Modify the equations of motion to include a linear damping term with damping coefficient β and an external forcing term of the form $f_0 \sin(\omega t)$.

What is the physical relevance of the following frequencies?

ω

$\sqrt{\frac{k}{m}}$

$\sqrt{\frac{k}{m} - \beta^2}$

$\sqrt{\frac{k}{m} - 2\beta^2}$

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