Name: ______________________________ Student number: ____________

Classical Mechanics
PHYB54, Final Exam

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University of Toronto, Scarborough

Thursday, April 11th 2017, 2pm-4pm

	Points	Max Points (+Bonus Points)
Question 1		4 (+1)
Question 2		3
Question 3		5
Question 4		8 (+4)
Question 5		6 (+3)
Question 6		5
Question 7		4 (+2)
Total		35 (+10)

- No aid sheets, books or other notes are allowed.
- All electronic devices must be stored together with your belongings at the back of the room.
- Write your answers on the question sheet. If you need more paper raise your hand.
- The University of Toronto's Code of Behaviour on Academic Matters applies to all University of Toronto Scarborough students. The Code prohibits all forms of academic dishonesty including, but not limited to, cheating, plagiarism, and the use of unauthorized aids. Students violating the Code may be subject to penalties up to and including suspension or expulsion from the University.

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Question 1 **4 Points (+1 Bonus Point)**

Define the term *inertial frame* and give two physical examples of non-inertial frames. You get one bonus point if your definition is shorter than 15 words.

Question 2 **3 Points**

Suppose that a projectile which is subject to a linear resistive force is thrown vertically down with a speed v_{y0} which is greater than the terminal speed v_{ter}. Describe and explain how the velocity varies with time, and make a plot of v_y against t for the case that $v_{y0} = 2v_{ter}$.

Question 3 **5 Points**

Highlight all forces which are conservative (k is a constant):

(a) $\vec{F} = k\ (x, 2y, 3z)$

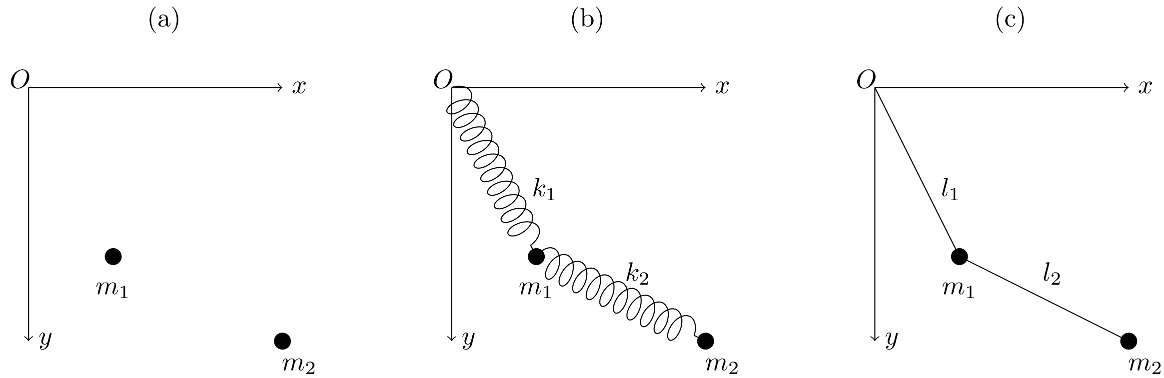
(b) $\vec{F} = k\ (y, x, 0)$

(c) $\vec{F} = k\ (-y, x, 0)$

For those which are conservative, find the corresponding potential energy U, and verify by direct differentiation that $\vec{F} = -\vec{\nabla} U$.

Question 4

8 Points (+4 Bonus Points)



Consider the above arrangements of two masses in two dimensions. In all cases, gravity is acting in the y direction. In figure (a), the masses can move completely freely. In figure (b), the masses are connected via a spring with spring constant k_2 and the mass m_1 is connected to the origin via a spring with spring constant k_1. In figure (c) the masses make up a double pendulum. Write down the Lagrangian for each case. You may use a coordinate system other than Cartesian, but you need to define it, i.e. with a sketch.

Case (c) is more difficult and counts towards bonus points.

$\mathcal{L}_a =$ ____

$\mathcal{L}_b =$ ____

$\mathcal{L}_c =$ ____

Use the Lagrangians from above and the Lagrange equations to calculate the equations of motions in each case. *Case (c) is more difficult and counts toward bonus points.*
a)

b)

c)

Question 5

6 Points (+3 Bonus Points)

Useful constants for this question:

Gravitational constant	$6.674 \cdot 10^{-11}\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Mass of the International Space Station (ISS)	$4.196 \cdot 10^5$ kg
Mass of the Earth	$5.972 \cdot 10^{24}$ kg
Radius of the Earth	$6.371 \cdot 10^6$m

What is the reduced mass of the Earth and the ISS (to within 4 significant digits)?

$\mu =$ ____________________________________

What is the rotation speed of the Earth at the equator in m/s?

$v_e =$ ____________________________________

The ISS is orbiting the Earth on a circular orbit at an altitude of 400 km above ground. Calculate the speed of the ISS relative to the centre of mass of the Earth.

$v_{\mathrm{iss}} =$ ____________________________________

What is the orbital period of the ISS in minutes?

$P_{\mathrm{iss}} =$ ____________________________________

Suppose we built a slightly larger rocket than the one from class, capable of sending a spacecraft from the surface of the Earth to the ISS. The initial thruster burn lasts just a few minutes and puts the spacecraft on an elliptical orbit. Assume the orbit's periapses is on the surface of the Earth and its

apoapsis reaches the orbit of the ISS. A second thruster burn is needed to put the spacecraft on the same circular orbit as the ISS.
Estimate (to within a factor of 2) the time in minutes after the launch at which the spacecraft's thrusters are turned on for the second time.

$t_{\text{burn}} =$ ________________________________

Bonus questions (3 points):
Estimate the velocity kick needed during the second thruster burn (to within a factor of 4):

$\Delta v_2 =$ ________________________________

Compare this velocity kick to that from the initial thruster burn (a rough estimate is ok).

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Question 6 **5 Points**

The potential energy of a mass m moving in one dimension (coordinate r) is

$$U(r) = \left(r + \frac{\lambda^2}{r}\right)$$

for $0 < r < \infty$, and λ being a positive constant. Find r_0 such that $\nabla U(r_0) = 0$ (this is the equilibrium position). Let x be the distance from equilibrium. Do a Taylor series expansion to second order in x such that the potential energy has the form $U = \text{const} + k\,x^2$. Use your result to calculate the angular frequency ω_0 for small oscillation around the equilibrium.

Assume we add a linear damping force of the form $F_d = -b\,\dot{r}$ and an external driving force of the form $F_e(t) = F_0 \sin(\omega t)$ to our system. Write down the long term evolution of $r(t)$. You don't need to write done explicit expressions for the amplitude and phase. Explain what happens when $\omega \sim \omega_0$. You may assume the amplitude remains small enough for the Taylor series expansion to be a good approximation.

Question 7

4 Points (+2 Bonus Point)

This question is about chaos. Mark all statements which are true.

- ☐ Being sensitive to small perturbations is the same as being chaotic.
- ☐ There are no periodic orbits in a chaotic system.
- ☐ Purely linear systems are never chaotic.
- ☐ A solution that shows period doubling is chaotic.
- ☐ The harmonic oscillator is chaotic.
- ☐ The double pendulum can be chaotic for large amplitudes.
- ☐ Stability is the opposite of chaos.

Bonus Question:

Is the Solar System chaotic? Is it stable? Discuss.

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