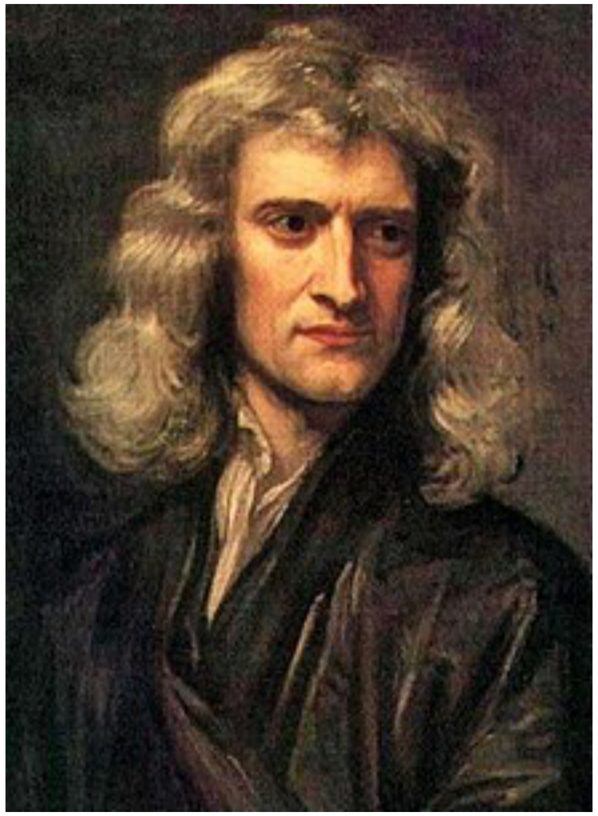
PHYB54 Revision



Isaac Newton
1642-1727

Not: Quantum Mechanics / Relativistic Mechanics

Classical mechanics breaks down if:

1) high speed, v ~ c
2) microscopic/elementary particles

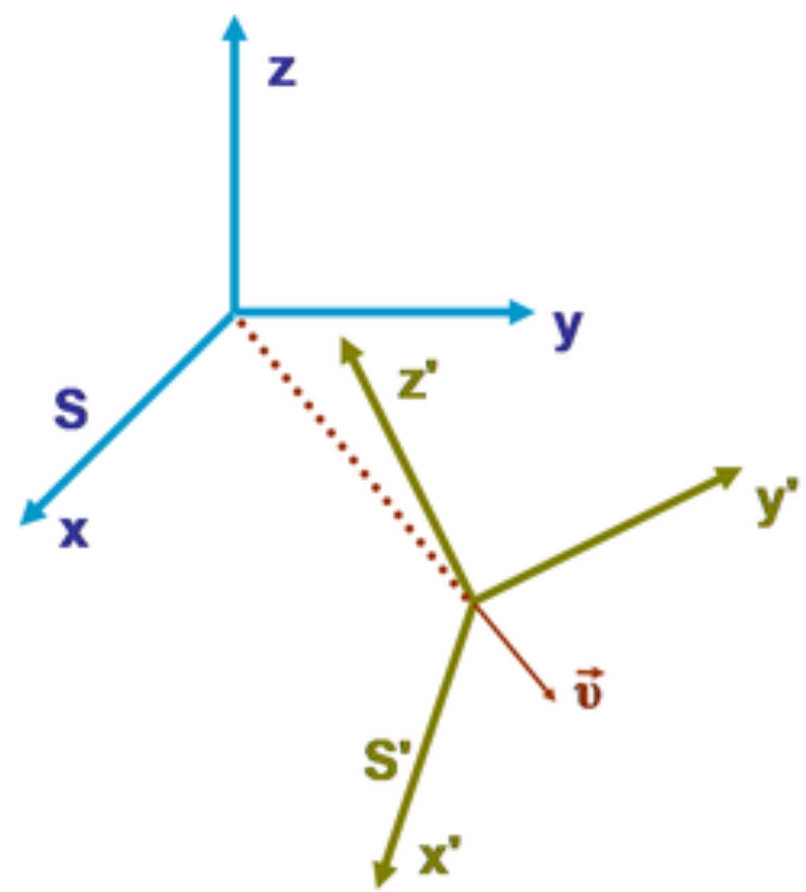
- Scalars, vectors
- Unit vectors
- Vector operations

Concept of time

Reference frames
- Not all frames are equal
- Be careful how you choose a frame

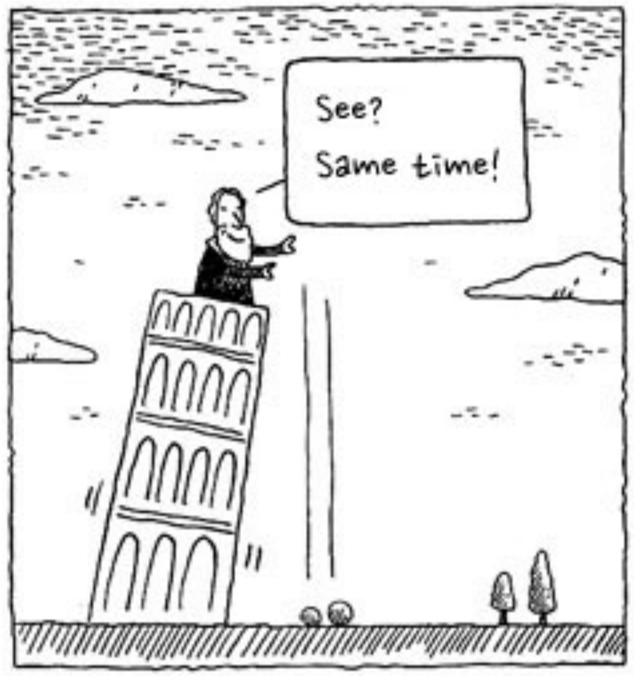
Inertial frames

- Newton’s laws define inertial frames (although not in the most obvious way).
- Simplest way of physical laws hold only in inertial frames
- Rotating or accelerating frames are not inertial



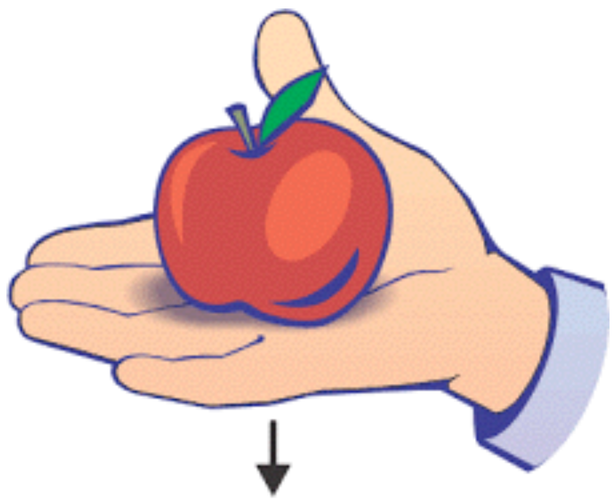
Mass

- Characterizes an object's inertia, its resistance to being accelerated.
- Define one unit of mass (kilogram)
- Then compare any other mass to it



- Mass is also proportional to weight
- Not obvious (Galileo)

Force
- Informal notion: a push or pull
- Define a unit of force: 1 Newton
- Force has direction



Point mass
- Has mass
- No size
- No internal degrees of freedom (rotation, vibration, deformation)

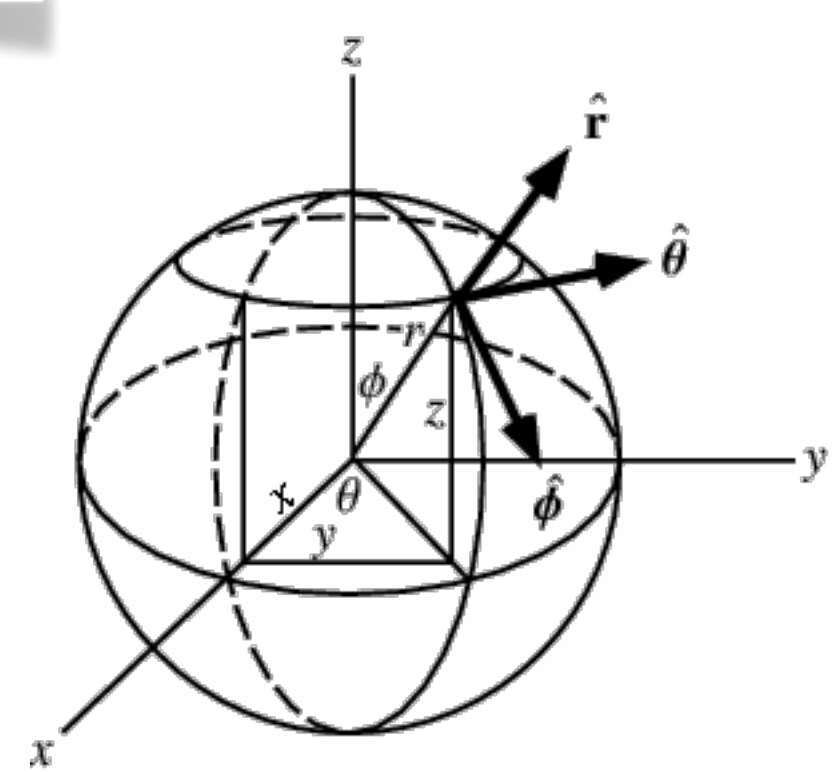
Newton’s First Law
In the absence of forces, a particle moves with constant velocity.

Newton’s Second Law
The net force on a particle is always equal to the mass times the particle’s acceleration.

Newton's Third Law
If object 1 excerpts a force F on object 2, then object 2 always excerpts a force -F on object 1.

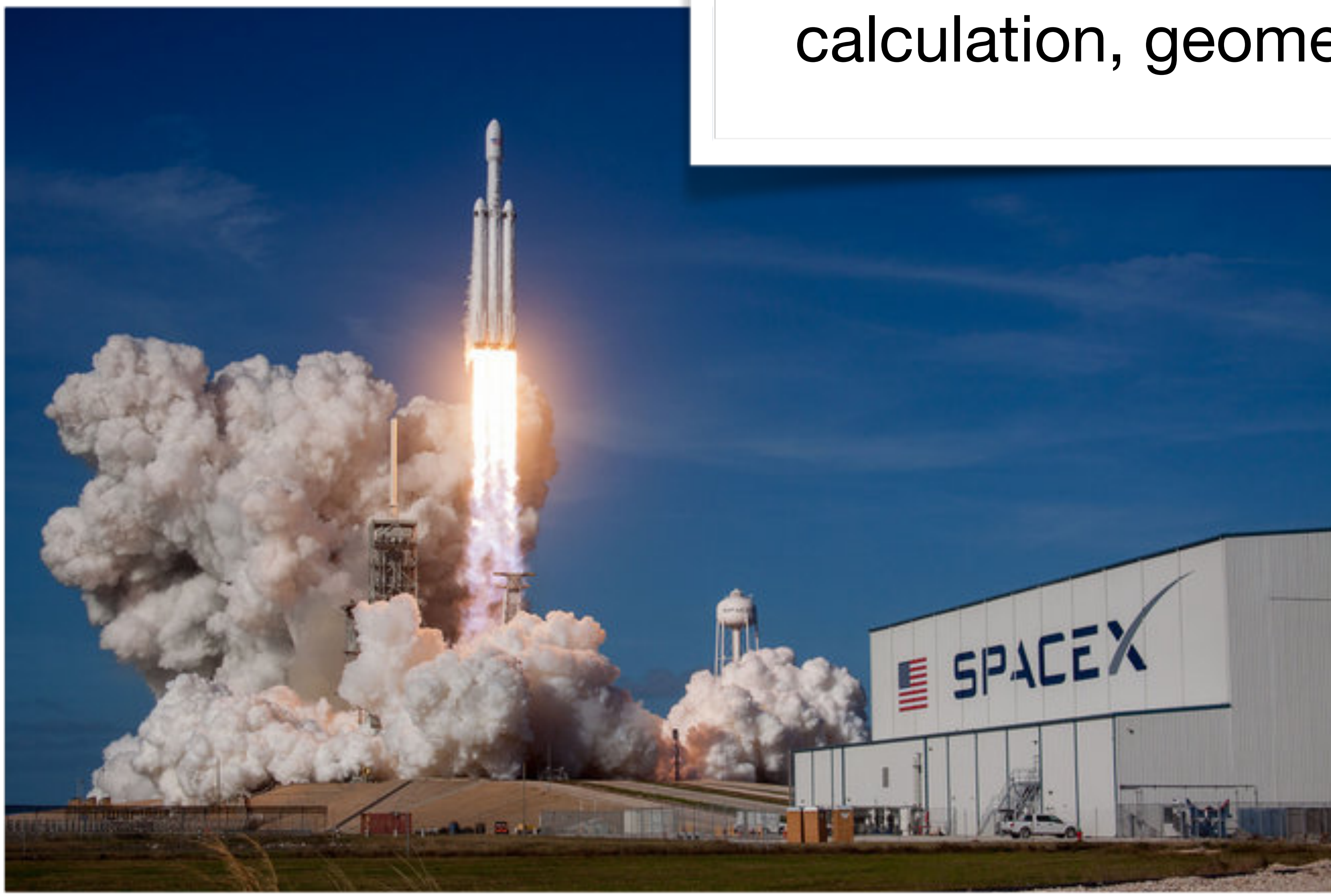
- Linked to conservation of momentum.
- Proof for conservation of momentum!

- Be comfortable doing calculations in polar and spherical coordinates
- In particular derivatives



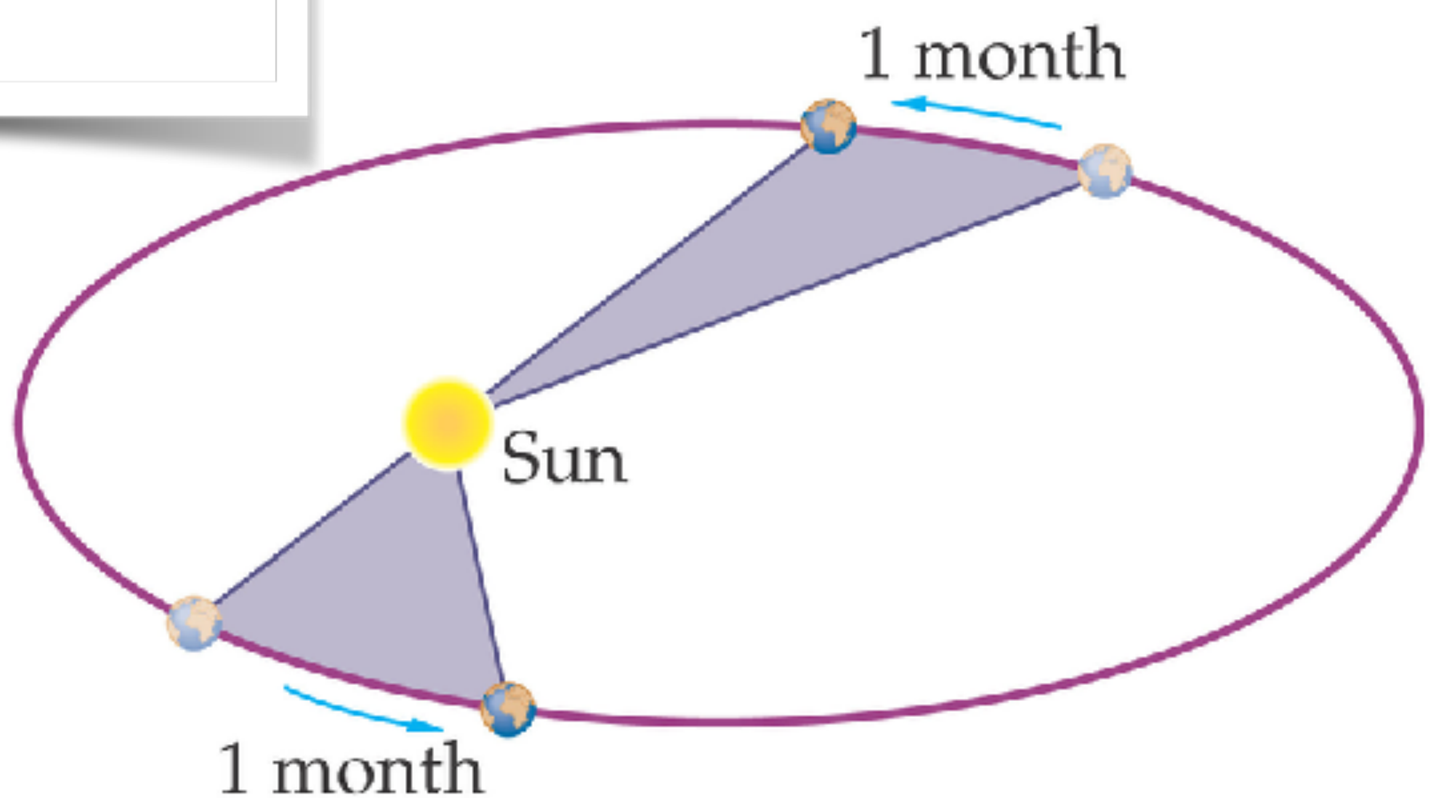
- Linear drag force
- Quadratic drag force
- Know physical origins
- How do the drag coefficients depend on size?
- Know which one matters for a given speed/size
- Be able to do simple calculations of projectiles with drag

- Proof of momentum conservation
- Rocket equation
- Centre of mass (definition, calculation, geometric construction)



- Proof of angular momentum conservation (one + multiple particles)
- What are the assumptions?

- Kepler's second law
- Proof!

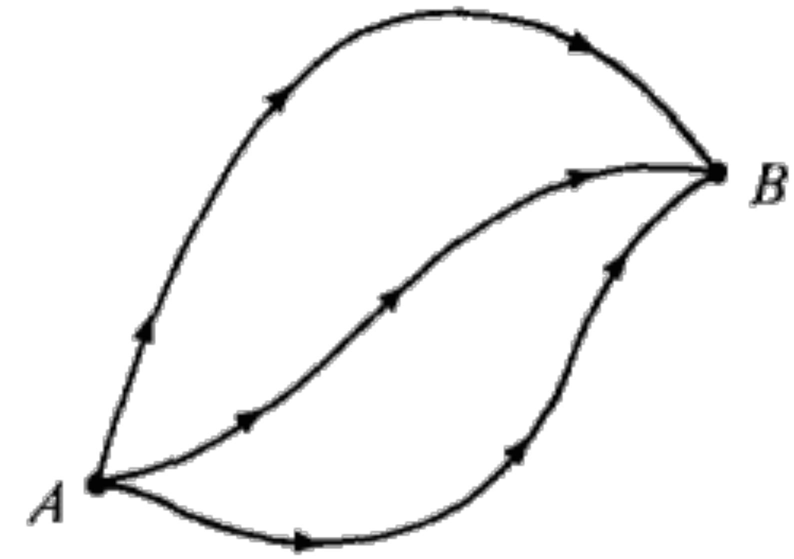


- Definition of kinetic energy
- Work-KE theorem in infinitesimal and integral form
- Derivation!

- Potential energy
- When can we use it?

- Conservation of energy

- Conservative forces
- What conditions need to be satisfied? (only depend on position, path independent)
- How to test path independence? Curl of force!



- Gradients in different coordinate systems

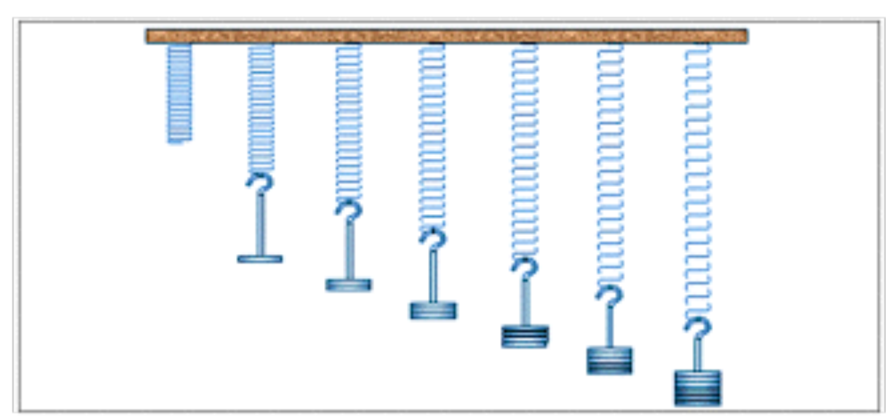
- Theorem: a central force is conservative iff it is spherically symmetric.

- Single potential describes forces between two interacting particles

- Energy of multi-particle systems
- Internal energy can be ignored
- Why is this important?

- Hooke's law
- Why is it so important?
- Taylor series expansion
- First term out of equilibrium that is important

- Know everything about the simple harmonic oscillator!
- Different solutions (complex exponential, sin+cos, sin+phase)



- Differential equation classification
- Order
- Linear / non-linear
- (non) homogeneous

1. $(1-x)y'' - 4xy' + 5y = \cos x$

2. $x\dfrac{d^3y}{dx^3} - \left(\dfrac{dy}{dx}\right)^4 + y = 0$

3. $t^5y^{(4)} - t^3y'' + 6y = 0$

4. $\dfrac{d^2u}{dr^2} + \dfrac{du}{dr} + u = \cos(r+u)$

5. $\dfrac{d^2y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

6. $\dfrac{d^2R}{dt^2} = -\dfrac{k}{R^2}$

7. $(\sin\theta)y''' - (\cos\theta)y' = 2$

8. $\ddot{x} - \left(1 - \dfrac{\dot{x}^2}{3}\right)\dot{x} + x = 0$

- Differential equation classification
- Order
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- (in) homogeneous

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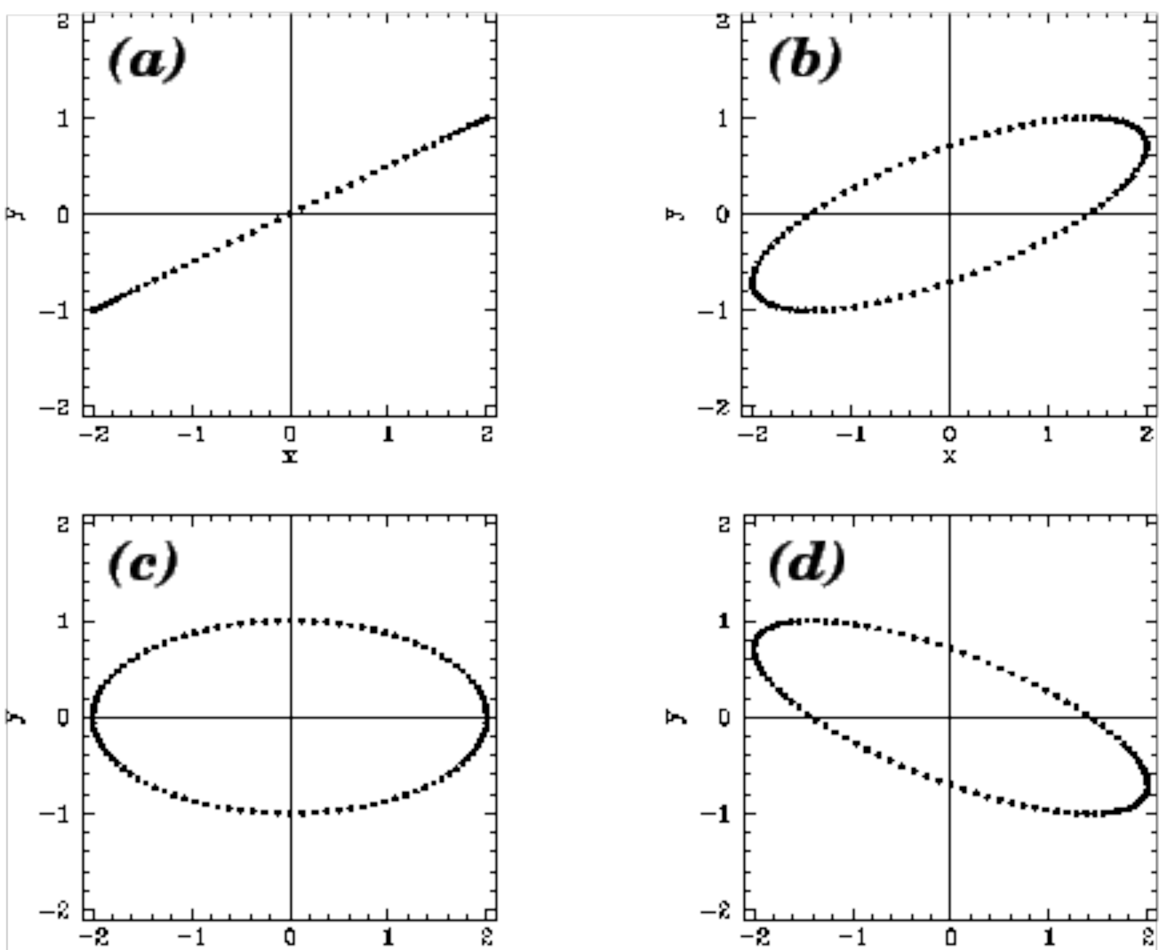
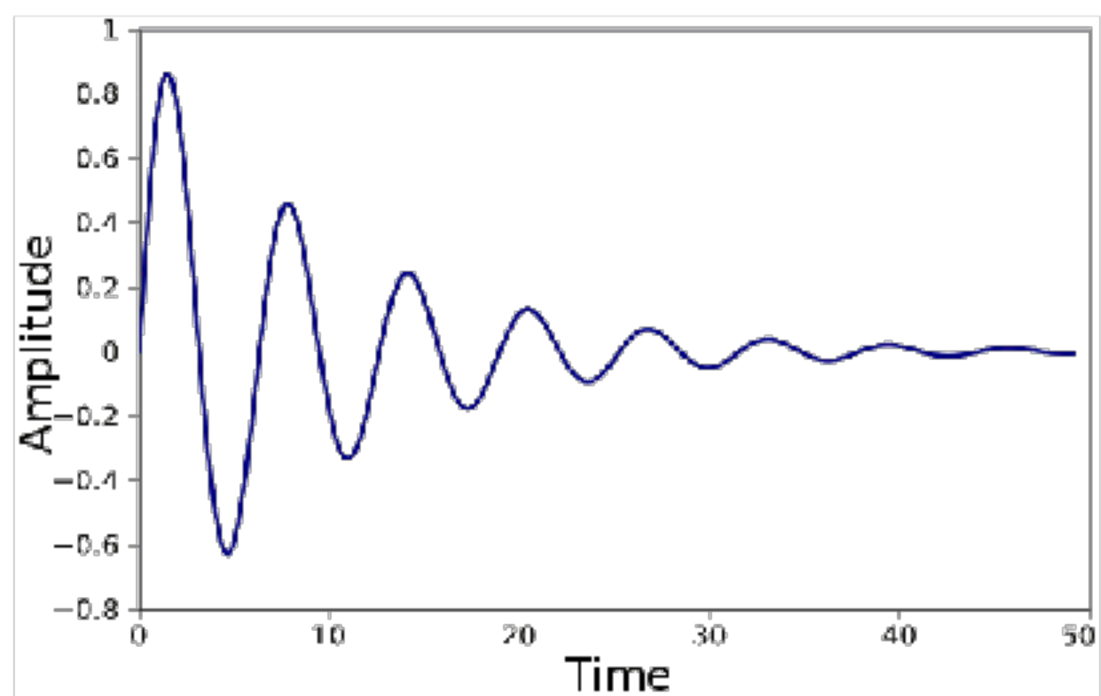
8. $\ddot{x} - \left(1 - \dfrac{\dot{x}^2}{3}\right)\dot{x} + x = 0$

2,4,5,6,8 are not linear

- Two dimensional isotropic oscillator
- Leads to uncoupled differential equations
- Know qualitative and quantitative solutions: no damping, critical damping, over damping, under damping

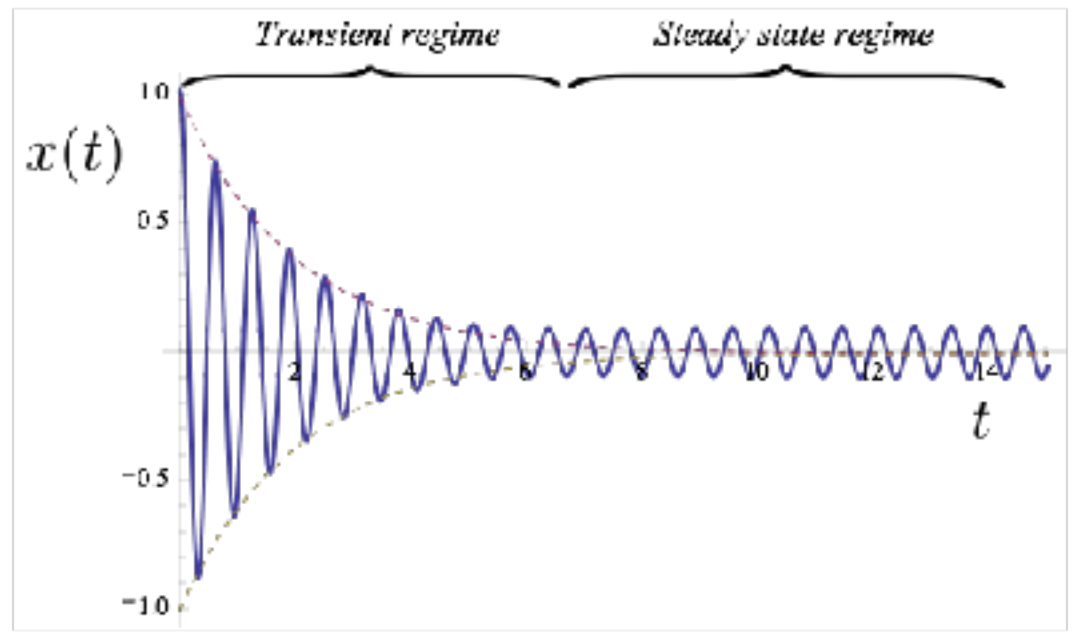


- Damped harmonic oscillator (linear damping)
- Know qualitative and quantitative solutions
- How to solve generic linear second order homogeneous differential equation? Find two independent solutions. Then any solution can be constructed as a linear combination.



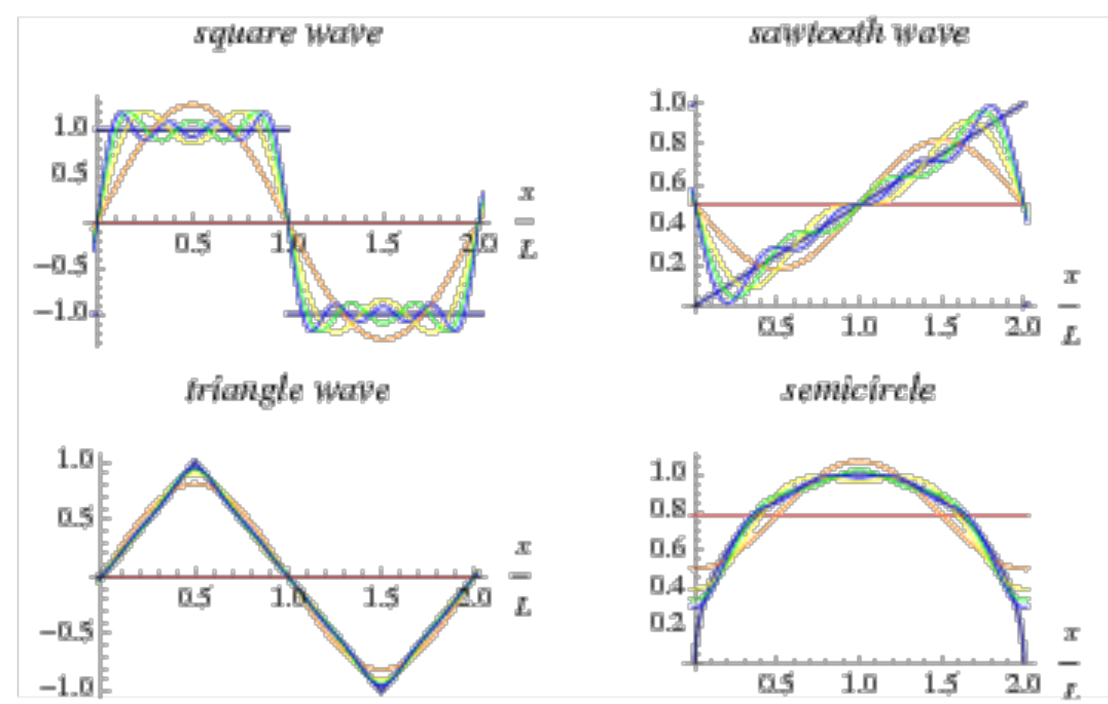
- Driven damped harmonic oscillator
- How to solve generic linear second order **in-**homogeneous differential equation? Find **one** solution. Then any solution can be constructed as a linear combination of homogenous solutions + the one in homogeneous solutions

- Transients and attractors
- Resonances!
- Amplitude of attractor solution increases near resonance



- Know different frequencies:
- Natural frequency
- Frequency of damped oscillator
- Frequency of driving force
- Frequency where response is maximal

- Fourier series
- Using it for the harmonic oscillator and the fact it is linear, lets us construct the response for any driving force

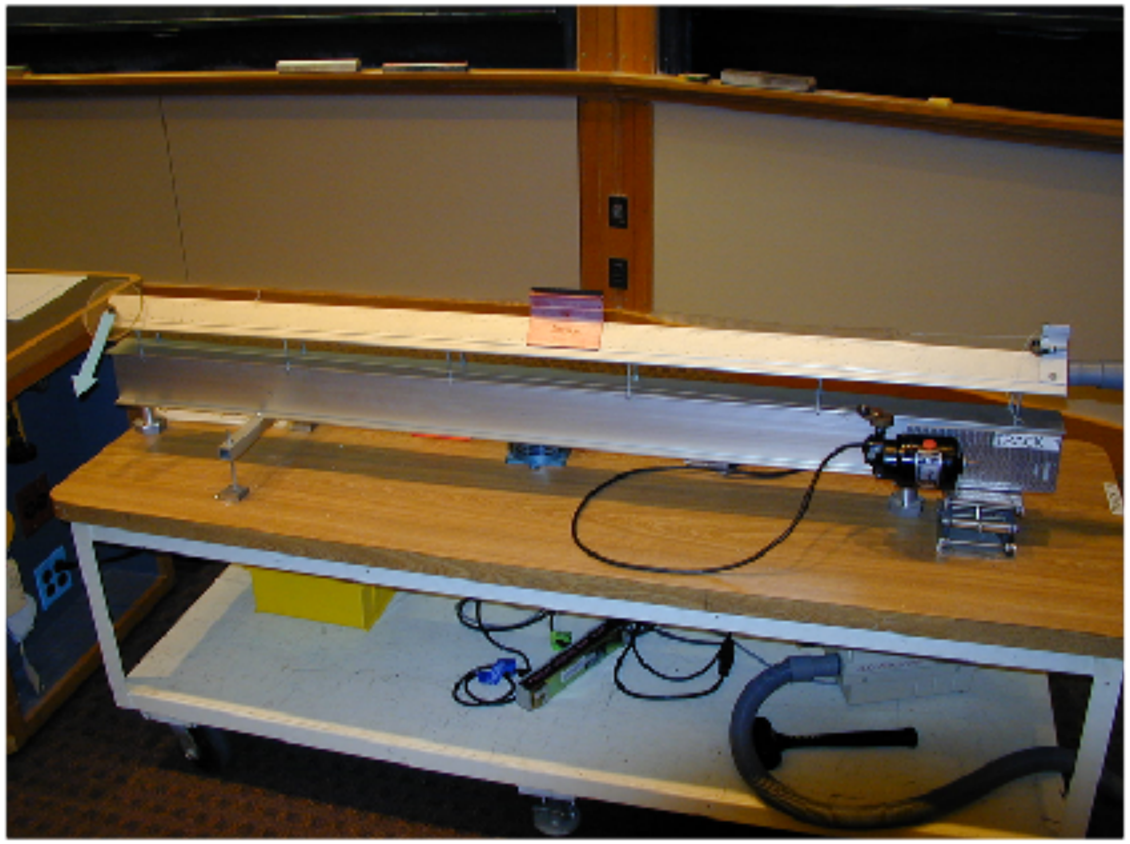


- New formalism, equivalent to Newton's laws
- Proof (in 1D, in inertial frame)
- What is the Lagrangian? What are the Lagrange Equations?

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_n}\right)-\frac{\partial L}{\partial q_n}=0$$

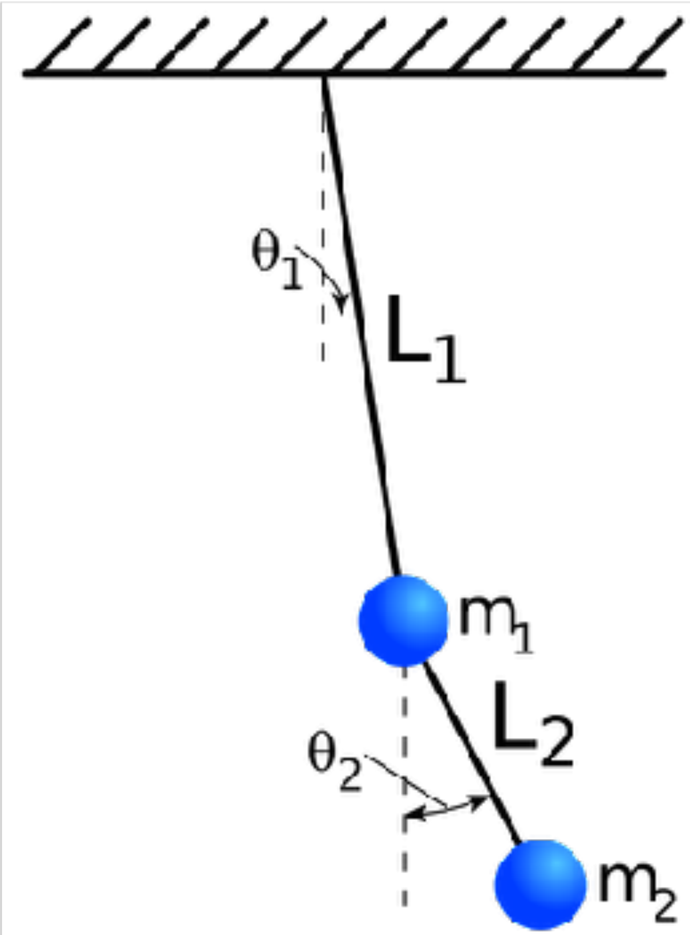
- Constrained systems
- Know two different way to solve them using Lagrangian
  1) Generalized coordinates
  2) Lagrange multipliers

- Be able to construct system of linear equations
- Bring it to normal matrix form
- Solve! Involves calculation of eigenvalue and eigenvectors.
- These are eigenfrequencies and eigenmodes
- Arbitrary solutions can be constructed as a linear combination of eigenmodes



- In particular, look at two cases:
- Equal mass, equal springs
- Weakly coupled

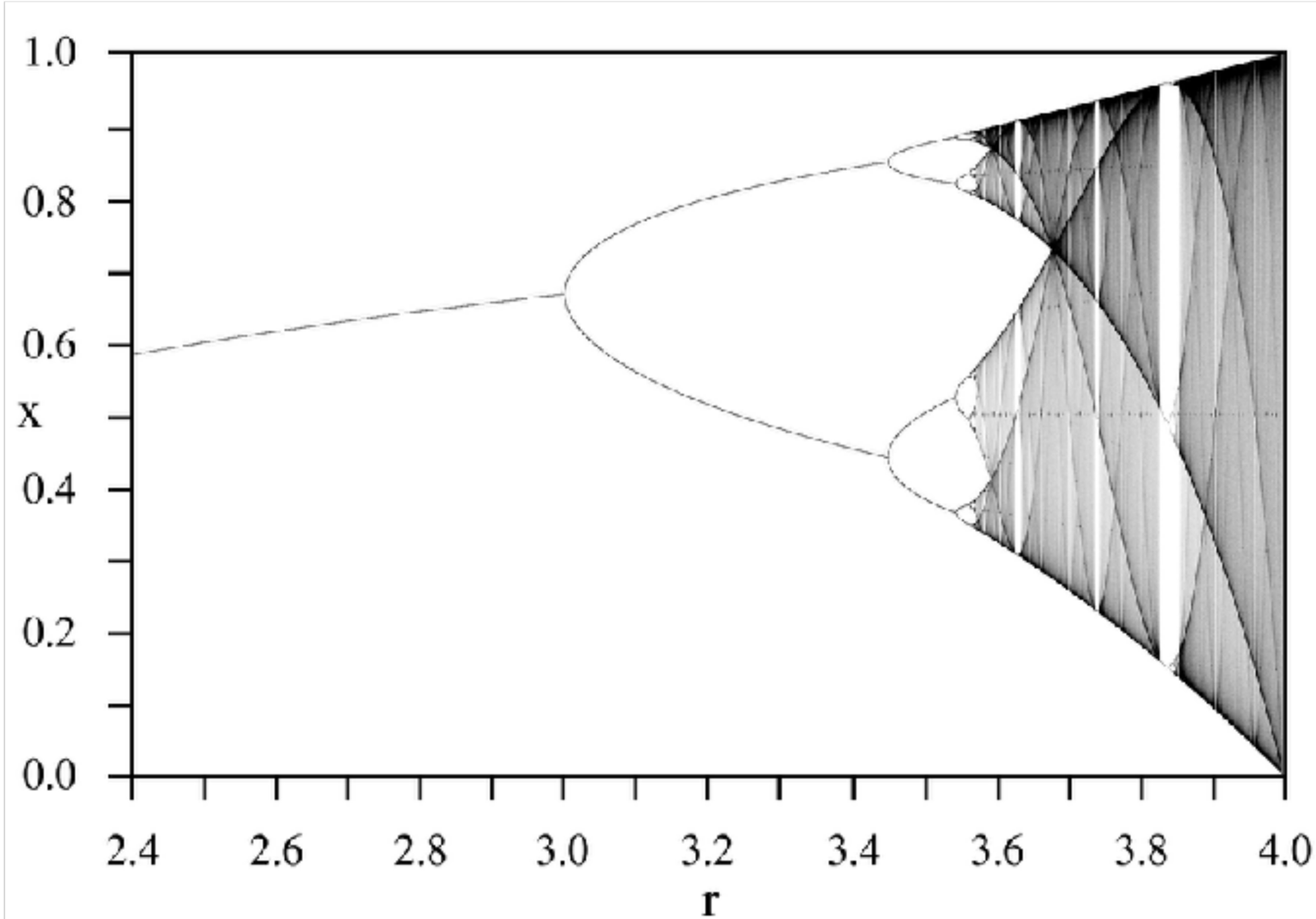
- Double pendulum
- Be able to write down Lagrangian in generalized coordinates
- How does solution for small angles look like?



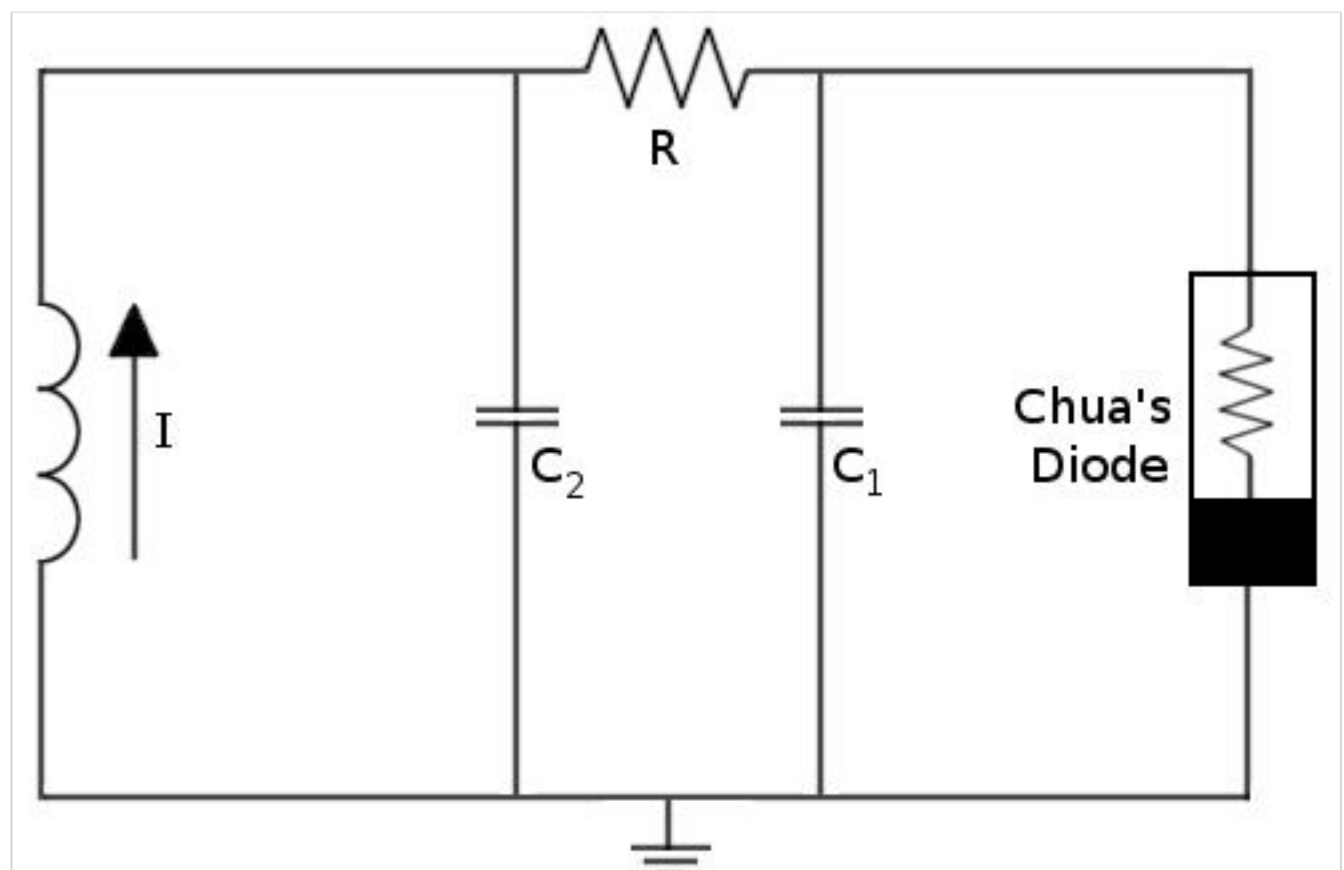
- Be able to characterize a differential equation
- What makes it non-linear?
- Look at the example of the damped driven pendulum (DDP)

1) Understanding chaos starts with understanding the linear case
2) Add non-linearities, expand in Taylor series
3) Harmonics appear
4) Going to higher and higher order, more harmonics
5) Eventually sub-harmonics appear (what is the difference?)
6) Period doubling cascade
7) After critical value, chaos
8) Sometimes islands of non-chaotic motion might exist

Understand the Bifurcation/
Feigenbaum diagrams!



- Know examples we discussed:
- DDP
- LC Circuit
- Chua’s circuit
- Double pendulum



Exam Format

- Focus will be on second half of the course
- But there will be some questions about the first half as well
- Look at the midterm again!
- You have two hours, which should be plenty of time
- You can use a non-programmable calculator and ruler
- But nothing else!
- Ask questions if you are unsure what I am asking for
- There will be bonus points, so you will not need to answer everything perfectly to get full marks