Classical Mechanics, PHYB54
Problem Set 6

Professor Hanno Rein
TA Ari Silburt

Due: Monday, March 13th 2017, 4pm

Note: Assignments can be hand-written, but illegible answers will not be marked. Clearly indicate your final answers.

Problem 6.1

Two particles of equal mass are moving on a frictionless surface with an interaction potential between them of $U_{12} = kr^2$, where $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ is the distance between the two particles. In addition, a central object exerts a potential of $U_1 = kr_1^2$ and $U_2 = kr_2^2$ on particles 1 and 2, respectively, where $\mathbf{r_1}$ and $\mathbf{r_2}$ are the distance from particles 1 and 2 from the central object.

(a) Write down the Lagrangian in terms of the Centre of Mass (CM) position $\mathbf{R}$, the relative position $\mathbf{r}$, the mass of particle 1 m_1 and constant k. The Lagrangian can then be split into CM and relative coordinate terms, $\mathcal{L} = \mathcal{L}_{CM} + \mathcal{L}_{rel}$ (see Equation 8.13 in the book).

(b) After performing this split and using Lagrange's equation, write down the equation of motion for the CM position $\mathbf{R}$.

(c) Similarly, using Lagrange's equation, write down the equation of motion in relative coordinates $\mathbf{r}$.

Problem 6.2

A planet is orbiting the Sun with aphelion distance r_{max}, perihelion distance r_{min}, initial eccentricity e_i and initial angular momentum ℓ_i. Keeping r_{min} fixed, the angular momentum ℓ is artificially increased over time.

(a) What happens to the eccentricity and r_{max}? Show calculations to support your claim.

(b) By what factor does the angular momentum need to be increased to make the planet's orbit unbound?

(c) Draw a few (at least 3) orbits showing how the orbit of the planet changes as ℓ increases. Clearly indicate which orbit corresponds to which value of ℓ.

Problem 6.3

A spacecraft in a circular orbit of radius R around a star wants to transfer to an elliptic orbit where perihelion is $0.2R$ and aphelion is at R. This maneuver is achieved by applying a single instantaneous thrust.

(a) What is the eccentricity of the elliptic orbit?

(b) How many times faster or slower is the spacecraft immediately after applying the instantaneous thrust? Is this an increase or decrease in speed?

(c) What is the difference in speed between aphelion and perihelion of the elliptical orbit? Is velocity slower or faster at aphelion?