Classical Mechanics, PHYB54
Problem Set 4

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You only have to answer the two out of the following four problems. I recommend you attempt problems 4.1 and 4.2.

Note: Assignments can be hand-written, but illegible answers will not be marked. Clearly indicate your final answers.

Problem 4.1

This question is about the collision of two particles. Assume that one particle has mass m_1 and speed v_1 and collides with a second particle of mass m_2 at rest.

(a) If the collision is perfectly inelastic (i.e. the particles stick together after the collision) what fraction of the kinetic energy is lost in the collision? Comment on the case of one of the masses being much larger than the other.

(b) As an application of thie problem, consider a slow and gentle collision between two asteroids. After a long time they are joined together. Given masses of $m_1 = 10^{12}$ kg and $m_2 = 4 \times 10^{12}$ kg and a relative velocity $v_1 = 50$ m/s, how much kinetic energy is converted to heat during the collision (assuming that all the kinetic energy lost is transferred to heat)? Assume these are metallic asteroids, made mostly of iron which will melt after the temperature has increased by 1600° Celsius. Using the specific heat capacity of iron, 450 J/kg and assuming that only 1/10 of each mass is heated up on the contact surface, are the asteroids welded?

Problem 4.2

This question is about the work-KE theorem of multiple particles. We consider a system of three particles here.

(a) Write down the work-KE theorem for each of the three particles separately and, by adding these three equations, show that the change in the total KE in a short time interval dt is $dT = W_{\text{tot}}$ where W_{tot} is the total work done on all particles by all forces.

(b) If we assume that for conservative forces $W_{\text{tot}} = -dU$ where dU is the change in total PE during the same time interval, comment on the total energy $E = T + U$. Explain in detail why this is logical.

Problem 4.3

The potential energy of a one-dimensional mass m at a distance r from the origin is

$$U(r) = U_0 \left[\frac{r}{R} + \lambda^2 \frac{R}{r} \right]$$

with $0 < r < \infty$, with U_0, R, and λ being positive constants.

(a) Find the equilibrium position r_0. Let x be the distance from equilibrium, and show that for small x the potential energy has the form of $U = constant + \frac{1}{2}kx^2$.

(b) What is the angular frequency of the small oscillations?

(c) The potential between molecules can be approximated using the Lennard-Jones potential which is

$$U(r) = 4\epsilon \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} \right]$$

What is the angular frequency for small oscillations in this potential well?

(d) Given that molecular hydrogen has vibrations given by the wavenumber of $4161.1663\,\mathrm{cm}^{-1}$ convert this to an angular frequency and then propose suitable values of ϵ and σ that could be used in a simulation. Provide a short justification for your proposed values.

Problem 4.4

The solution for $x(t)$ for a driven, underdamped oscillator is

$$x(t) = A\cos(\omega t - \delta) + e^{-\beta t}[B_1 \cos(\omega_1 t) + B_2 \sin(\omega_1 t)]$$

.

Solve that equation and the corresponding expression for $\dot{x}$, to give the coefficients B_1 and B_2 in terms of A, δ, and the initial conditions x_0 and v_0. That is, verify that $B_1 = x_0 - A\cos(\delta)$ and $B_2 = \frac{1}{\omega_1}(v_0 - \omega A \sin(\delta) + \beta B_1)$.