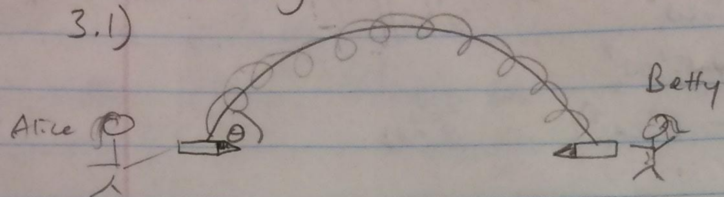
Assignment 3 Solutions - 10 points total

3.1)



$y = v_{0y}t + \frac{1}{2}at^2$

$0 = v_0 \sin\theta\, t - \frac{1}{2}gt^2$

$\boxed{t = \frac{2v_0 \sin\theta}{g}}$ → 1 point

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $\alpha = 0$, no net acceleration on angular velocity

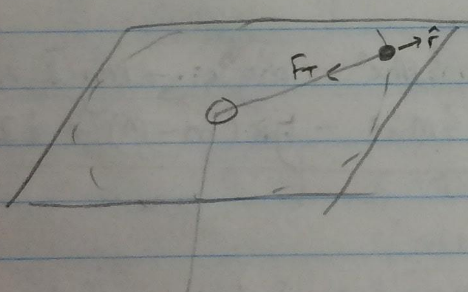
$\theta = \omega_0 t$

$2\pi(n+0.5) = \omega_0 t$

$\frac{2\pi(n+0.5)}{t} = \omega_0$

$\boxed{\omega_0 = \frac{\pi g(n+0.5)}{v_0 \sin\theta}}$ → 1 point

3.2)



a) angular momentum is conserved:

$L_f = L_i$

$\cancel{m} v_f r_f = \cancel{m} v_i r_i$

$\omega_f r_f^2 = \omega_i r_i^2$

$\boxed{\omega_f = \omega_i \left(\frac{r_i}{r_f}\right)^2}$ → 1 point

b) $W = \int \vec{F} \cdot d\vec{r} = \int \vec{F}_T \cdot d\vec{r}$ → F_T and dr face opposite directions.

$= -\int F\, dr$

$F = ma = \frac{mv^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r = \frac{m\omega_i^2 r_i^4 r}{r^4}$

$F = \frac{m\omega_i^2 r_i^4}{r^3}$ → 1 point

$$W = -\int F\,dr = -m\omega_i^2 r_i^4 \int_{r_i}^{r_f} \frac{dr}{r^3} = -m\omega_i^2 r_i^4 \left[\frac{-1}{2r^2}\right]_{r_i}^{r_f}$$

$$\boxed{W = \frac{m\omega_i^2 r_i^4}{2}\left(\frac{1}{r_f^2} - \frac{1}{r_i^2}\right)} \rightarrow \text{1 point}$$

c) $\Delta KE = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2$

$= \frac{1}{2} m \cancel{r_f^2} \frac{\omega_i^2 r_i^4}{\cancel{r_f^4}} - \frac{1}{2} m r_i^2 \omega_i^2$

$$\boxed{\Delta KE = m\omega_i^2 r_i^4 \left(\frac{1}{r_f^2} - \frac{1}{r_i^2}\right) = \text{Work done}} \rightarrow \text{1 point}$$

d) $KE_f \overset{?}{>} KE_i$

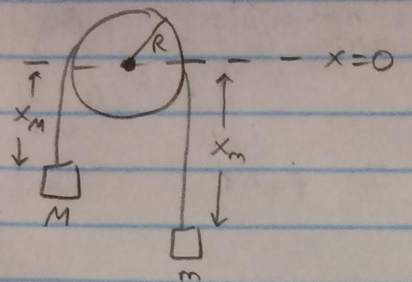
$\cancel{\tfrac{1}{2}} I_f \omega_f^2 \overset{?}{>} \cancel{\tfrac{1}{2}} I_i \omega_i^2$

$\cancel{m} \cancel{r_f^2} \frac{\cancel{\omega_i^2} r_i^4}{r_f^4} \overset{?}{>} \cancel{m} \cancel{r_i^2} \cancel{\omega_i^2}$

$\boxed{\frac{r_i^2}{r_f^2} \overset{?}{>} 1}$ → Since Prof. Rein pulled the string in, $r_f < r_i$, and this is a true statement. Thus, original statement also true: $KE_f > KE_i$

1 point

3.3)



substitutions

$x_M = \ell - x_m$, ℓ = total length of string, constant

$\dot{x}_M = -\dot{x}_m$

$\dot{x}_M^2 = \dot{x}_m^2$

$\omega = V/R = \dot{x}/R$

a) $E = \frac{1}{2}m\dot{x}_M^2 + \frac{1}{2}m\dot{x}_m^2 - mgx_m - Mgx_M + \frac{1}{2}I\omega^2$

fill in substitutions

$E = \frac{1}{2}m\dot{x}_m^2 + \frac{1}{2}m\dot{x}_m^2 - mgx_m - Mg(\ell - x_m) + \frac{1}{2}I(\dot{x}/R)^2$

drop subscript

$$\boxed{E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}^2 - mgx - Mg(\ell - x) + \frac{1}{2}I(\dot{x}/R)^2} \rightarrow \text{1 point}$$

b) It is a conservative system since the only force acting is gravity and its curl $\vec{\nabla} \times \vec{F} = 0$.

1 point

c) Since it's a conservative system, total energy is constant:

$$\frac{dE}{dt} = 0 = M\ddot{x} + m\ddot{x} - mg - Mg(-1) + \frac{I}{R^2}\ddot{x}$$

$$\ddot{x}(M + m + I/R^2) = g(m - M)$$

$$\boxed{\ddot{x} = \frac{g(m-M)}{(M + m + I/R^2)}} \rightarrow \text{1 point}$$