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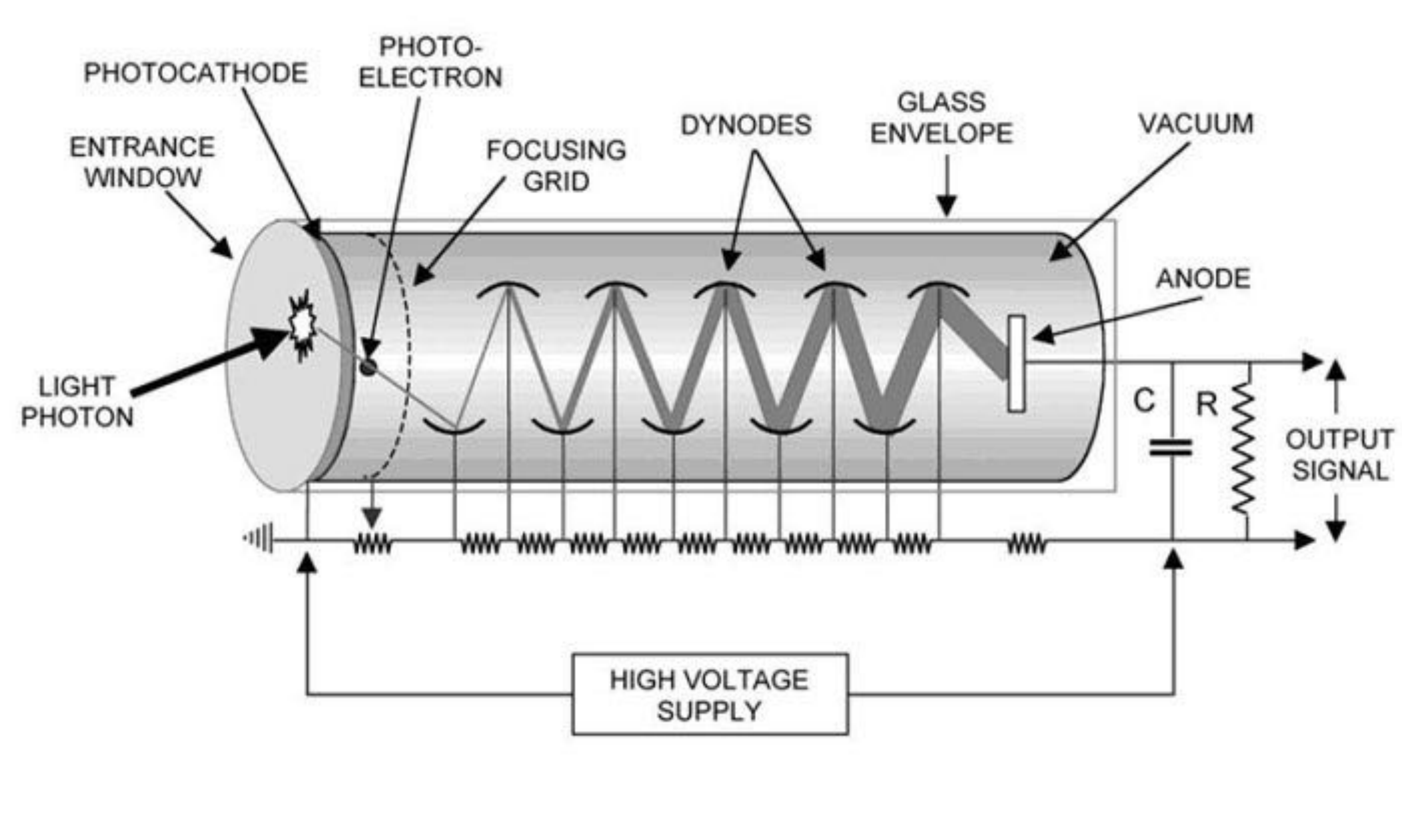
DETECTORS

BEFORE CCDS

Photographic plates for images

Photomultiplier tubes for photometry

PHOTOMULTIPLIER TUBES



PHOTOMULTIPLIER TUBES

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STATISTICS

STATISTICS

Precision with which a quantity is measures is important.

In this lecture: Frequentist approach

Better: Bayesian approach

DIFFERENT SOURCES OF NOISE
INSTRUMENTAL NOISE

Every instrument has background noise.

Need to measure the background noise, then subtract it.

Note that measuring the background noise has its own error associated with it!

INSTRUMENTAL NOISE

SYSTEMATIC

E.g. temperature changes

Unpredictable manner.

Can NOT be improved by more measurements.

STATISTICAL

E.g. random cosmic rays

Can be measured as accurately as desired.

But it may take a lot of time.

STATISTICAL FLUCTUATIONS

Inherent randomness of events

EXAMPLE: PHOTONS FROM A STAR (POISSON STATISTICS)

Well defined average number of photons that arrive on detector, 100 photons per second

If measured for finite time, will fluctuate from interval to interval

Assumption: all events are uncorrelated

Get distribution of counts N(x): 105, 98, 87, 96, 103, 101, 97

EXAMPLE: PHOTONS FROM A STAR (POISSON STATISTICS)

Distribution of counts N(x): 105, 98, 87, 96, 103, 101, 97

$$P_x = \frac{m^x e^{-m}}{x!}$$

Probability of detecting x events (integer) if the average is m.

If we collect 100 photons in a pixel on average, how likely is it that we collect exactly 100 in a single frame?

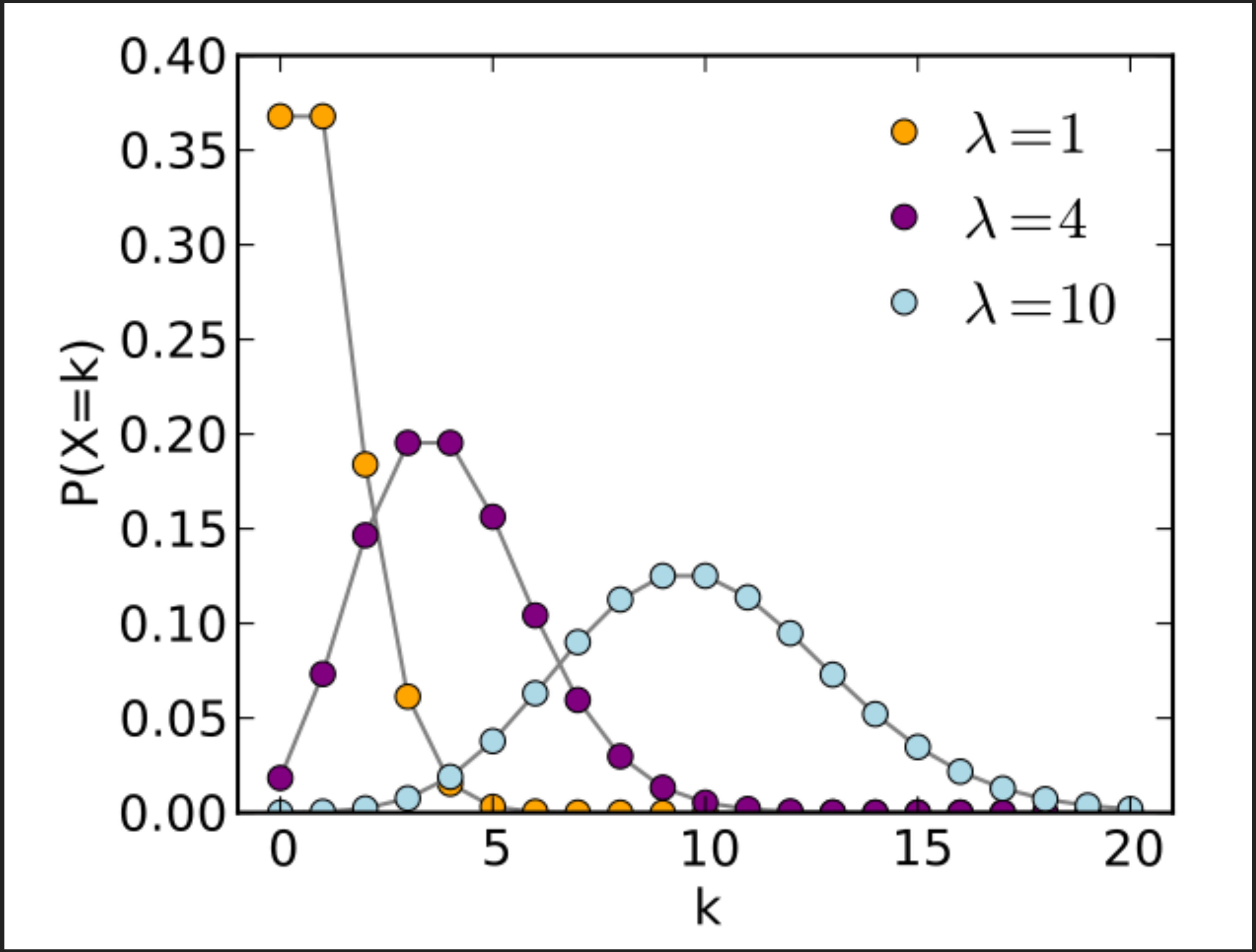
If we collect 6 photons in a pixel on average, how likely is it that we collect exactly 6 in a single frame?

EXAMPLE: PHOTONS FROM A STAR (POISSON STATISTICS)

$$P_x = \frac{m^x e^{-m}}{x!}$$

Note that:

$$\sum_{x=0}^{\infty} P_x = 1$$



NORMAL DISTRIBUTION

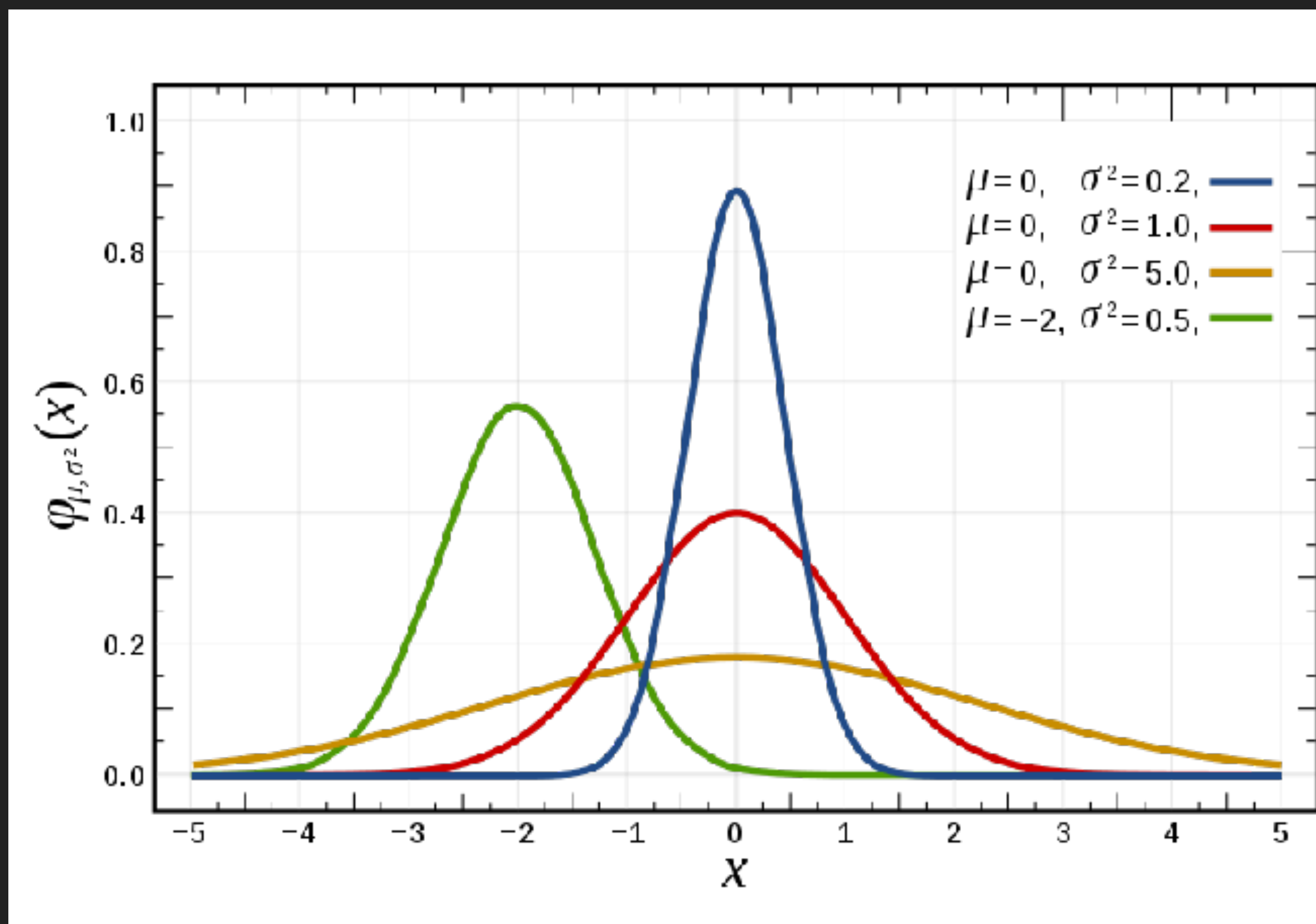
$$dP_x = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

$$\int_{x=-\infty}^{x=+\infty} dP_x = 1$$

- Continuous –> differential probability
- Two parameters, m and σ
- Symmetric
- Will give negative values!

NORMAL DISTRIBUTION

$$dP_x = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

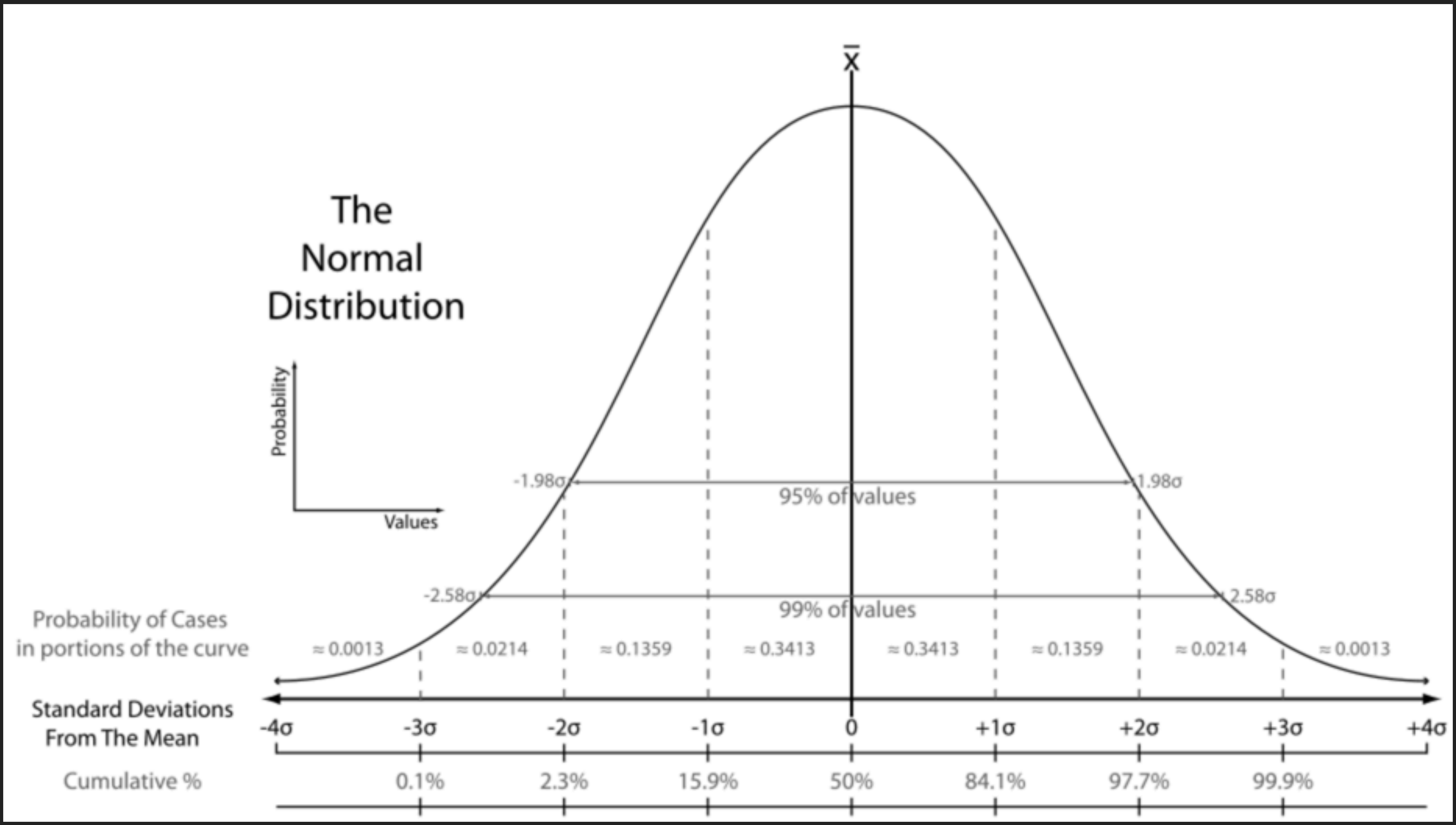


NORMAL DISTRIBUTION

$$P_x = \frac{m^x e^{-m}}{x!}$$

$$dP_x = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

- For large m Poisson distribution -> normal distribution
- $\sigma = \sqrt{m}$
- For a lot of the events that we measure m is so large, we use the normal distribution even though a Poisson distribution would be more appropriate


x̄
The Normal Distribution
Probability
Values
-1.98σ
1.98σ
95% of values
-2.58σ
2.58σ
99% of values
Probability of Cases in portions of the curve
≈ 0.0013
≈ 0.0214
≈ 0.1359
≈ 0.3413
≈ 0.3413
≈ 0.1359
≈ 0.0214
≈ 0.0013
Standard Deviations From The Mean
-4σ
-3σ
-2σ
-1σ
0
+1σ
+2σ
+3σ
+4σ
Cumulative %
0.1%
2.3%
15.9%
50%
84.1%
97.7%
99.9%

VARIANCE AND STANDARD DEVIATION

- Definitions are valid for any distribution
- Variance:

$$\sigma^2 = \frac{1}{n}\sum_{i=0}^{n}(x_i - m)^2$$

- Standard deviation σ
- For normal distribution, $\sigma = \sigma$
- Practical variance (because m is not independent)

$$\sigma^2 = \frac{1}{n-1}\sum_{i=0}^{n}(x_i - x_{av})^2$$

EXAMPLE

- On average, we have m=100 photons on the CMOS sensor per s
- Poisson distribution
- 1s exposure: $\sigma = \sqrt{100} = 10$ $\sigma / m = 0.1$
- 100s exposure: $\sigma = \sqrt{10000} = 100$ $\sigma / m = 0.01$

EXAMPLE

- In reality, we are not photon limited
- Other noise sources dominate, including:
  - Thermal noise
  - CMOS Amplifier
  - Atmosphere
- Will assume normal distribution (note: there is a better way)
- Can use multiple measurement to estimate mean and variance

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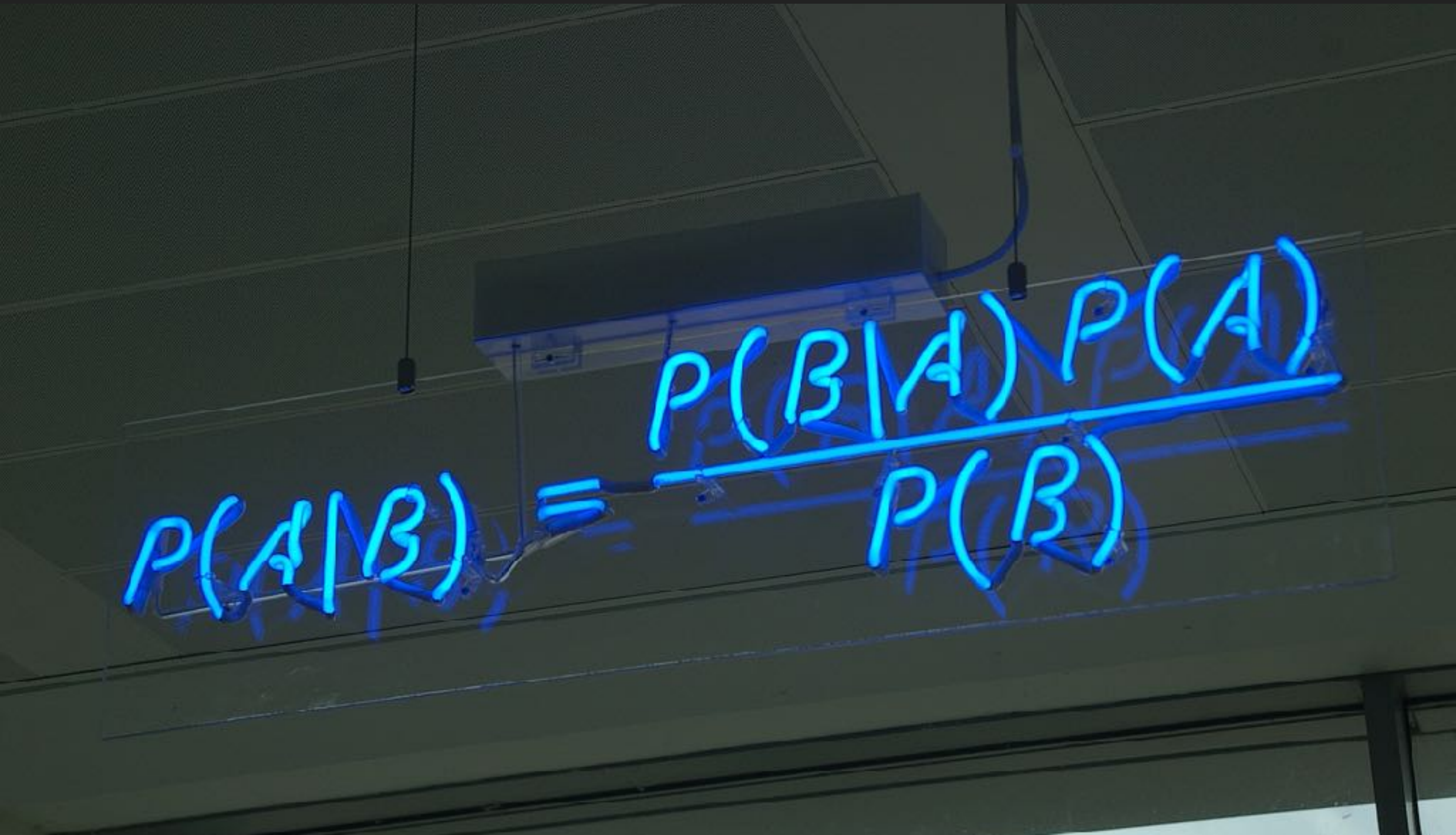
MCMC

BAYES'S THEOREM



DIACHRONIC INTERPRETATION

One way of thinking about Bayes's theorem. Suppose we have a hypothesis H and some data D.

$$p(H|D) = \frac{p(H)\ p(D|H)}{p(D)}$$

These terms now have names and can be interpreted as follows.

DIACHRONIC INTERPRETATION

$$p(H|D) = \frac{p(H)\ p(D|H)}{p(D)}$$

$p(H)$ Prior

$p(H|D)$ Posterior

$p(D|H)$ Likelihood

$p(D)$ Normalization constant

Monty Hall Problem

- An application of Bayes's theorem
- Player picks door 1, host opens door 2
- Should the player change their initial choice?
- A = car is behind door 1
  B = host opens door 2
  C = car is behind door 3

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \qquad P(C \mid B) = \frac{P(B \mid C)P(C)}{P(B)}$$

How to calculate P(B)

- Involves summation over all possible scenarios

- In our case A and C:

$$P(B) = P(B|A)P(A) + P(B|C)P(C)$$

- P(B) not needed to decide to decide if we should open door 1 or 3

LINE FITTING

$p(D|H)$ Likelihood

One data point:

$$p(y_i|x_i,\sigma_{yi},m.b) = \frac{1}{\sqrt{2\pi\sigma_{yi}^2}}\exp\left(-\frac{(y_i - mx_i - b)^2}{2\sigma_{yi}^2}\right)$$

Multiple data points:

$$\mathcal{L} = \prod_{i=1}^{N} p(y_i|x_i,\sigma_{yi},m,b)$$

LINE FITTING

Multiple data points:

$$\mathcal{L} = \prod_{i=1}^{N} p(y_i|x_i, \sigma_{yi}, m, b)$$

Log likelihood:

$$\ln \mathcal{L} = K - \sum_{i=0}^{N} \frac{(y_i - mx_i - b)^2}{2\sigma_{yi}^2}$$
$$= K - \frac{1}{2}\chi^2$$

BAYES THEOREM

$$p(m, b| \{y_i\}_{i=1}^N, I) = \frac{p(\{y_i\}_{i=1}^N |m, b, I)\; p(m, b|I)}{p(\{y_i\}_{I=1}^N |I)}$$

I Short hand for all prior knowledge

$\{y_i\}_{i=1}^N$ Short hand for all data

BAYES THEOREM

$$p(m,b|\left\{y_i\right\}_{i=1}^{N},I) = \frac{p(\left\{y_i\right\}_{i=1}^{N}|m,b,I)\;p(m,b|I)}{p(\left\{y_i\right\}_{I=1}^{N}|I)}$$

We want to know this!
It's a distribution!

We can *sample* the distribution!

BAYES THEOREM

$$p(m,b|\left\{y_i\right\}_{i=1}^N,I) = \frac{p(\left\{y_i\right\}_{i=1}^N|m,b,I)\;p(m,b|I)}{p(\left\{y_i\right\}_{I=1}^N|I)}$$

Let's ignore the normalization constant (does not depend on m or b)

METROPOLIS HASTINGS ALGORITHM

1) Start at a random place (ideally close to a sensible value)
2) Predict random new place
3) a) Choose new place if it’s better
   b) ‘Sometimes’ choose even if it’s worse
4) Keep track of path. Path is the posterior distribution!

METROPOLIS HASTINGS ALGORITHM

We need:

1) Likelihood/Prior function
2) MH algorithm
3) Starting point