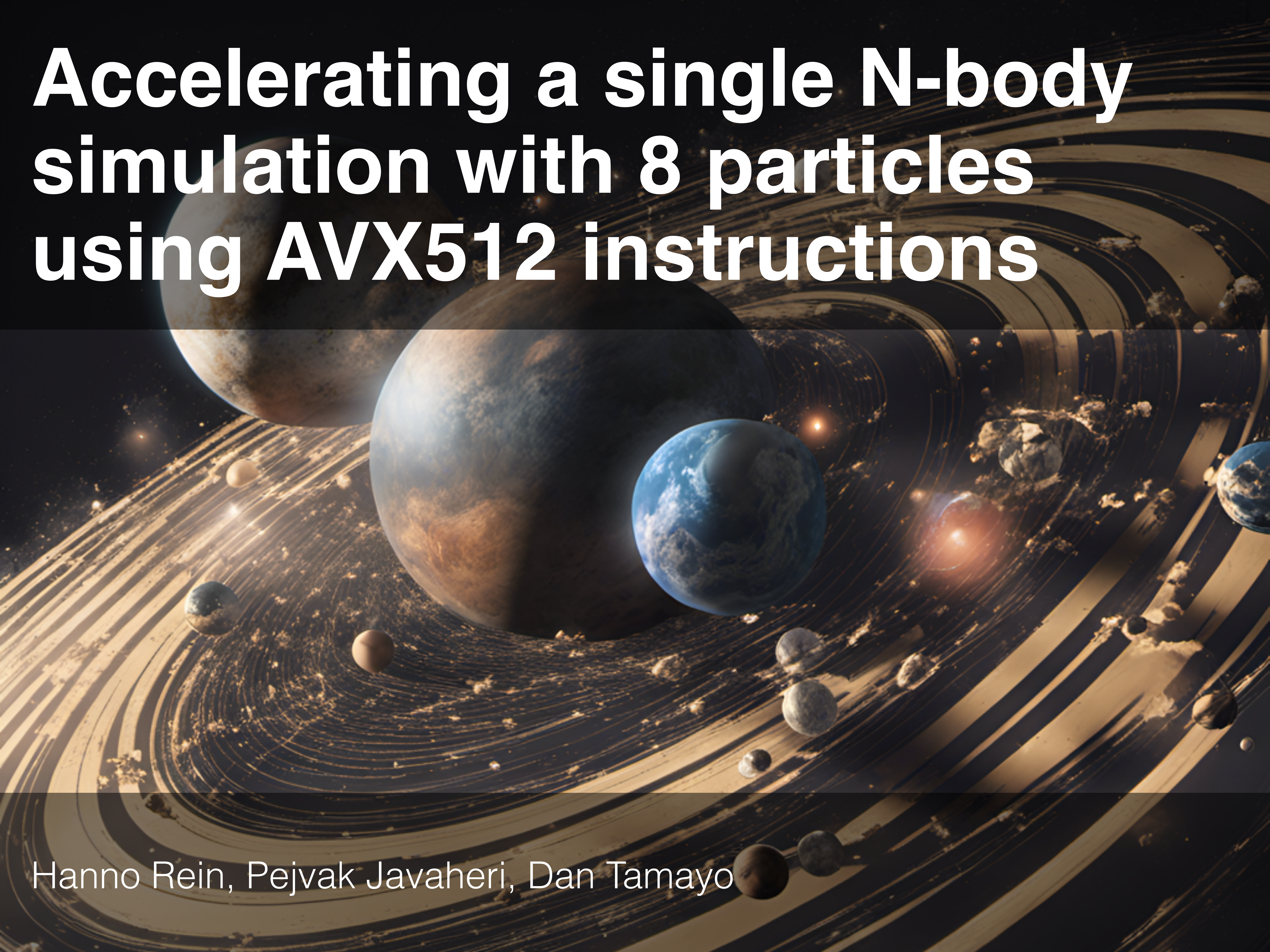


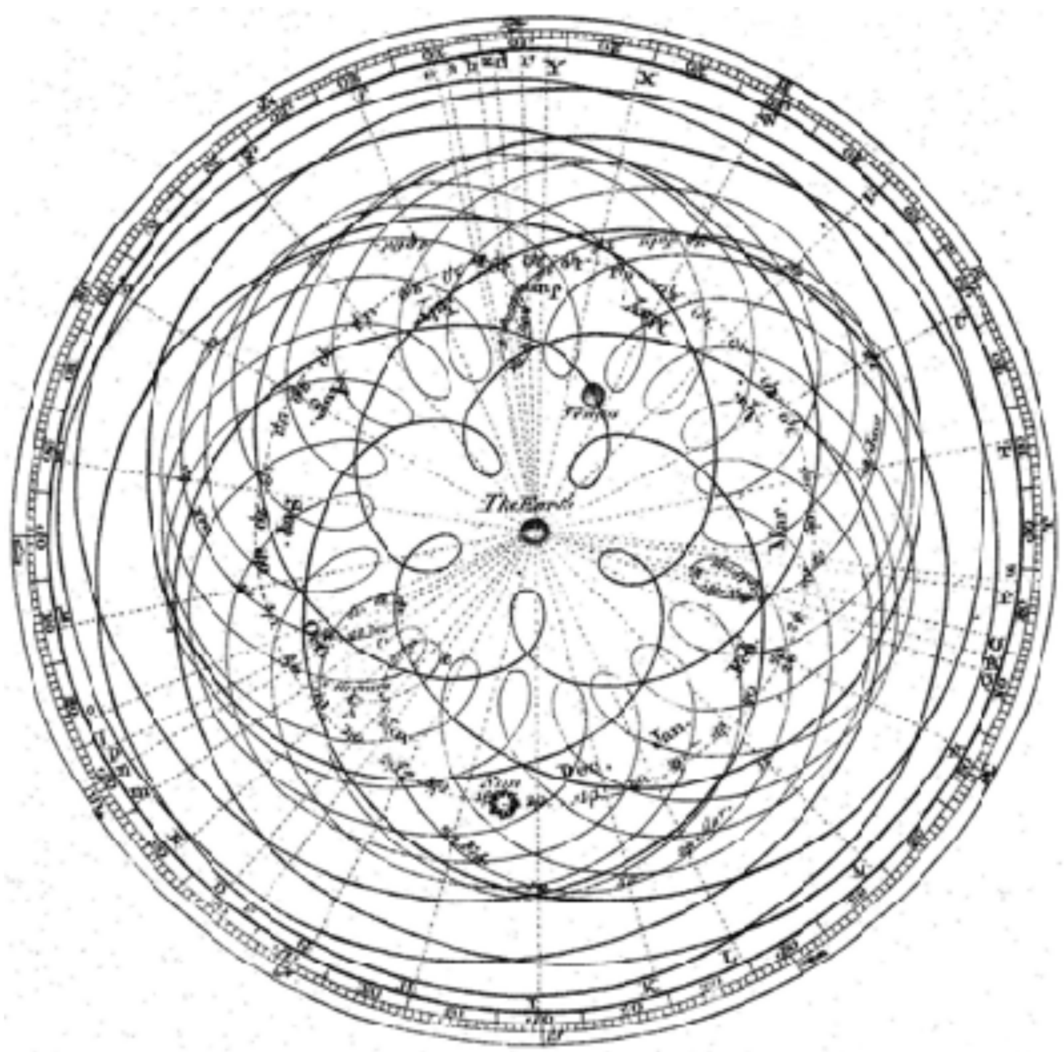
Accelerating a single N-body simulation with 8 particles using AVX512 instructions

Hanno Rein, Pejvak Javaheri, Dan Tamayo

Newton (1687)

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}$$

N-body simulations of planetary systems



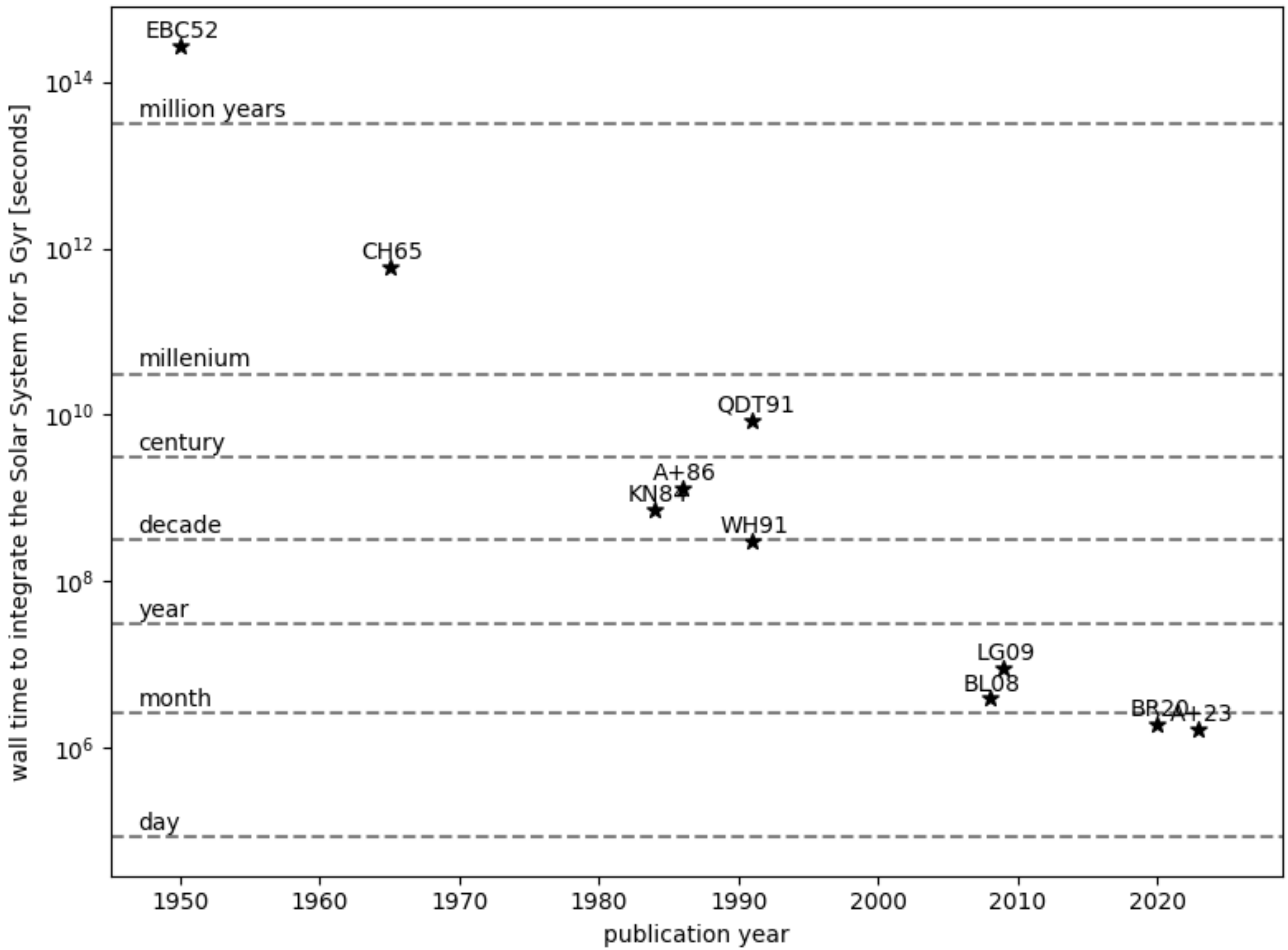
- Calculate orbits of planets, asteroids, and comets in the Solar System
- Find out if a planetary system is stable
- Hard to do for the Solar System
- Even harder for exoplanetary systems

Why is it hard?

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}$$

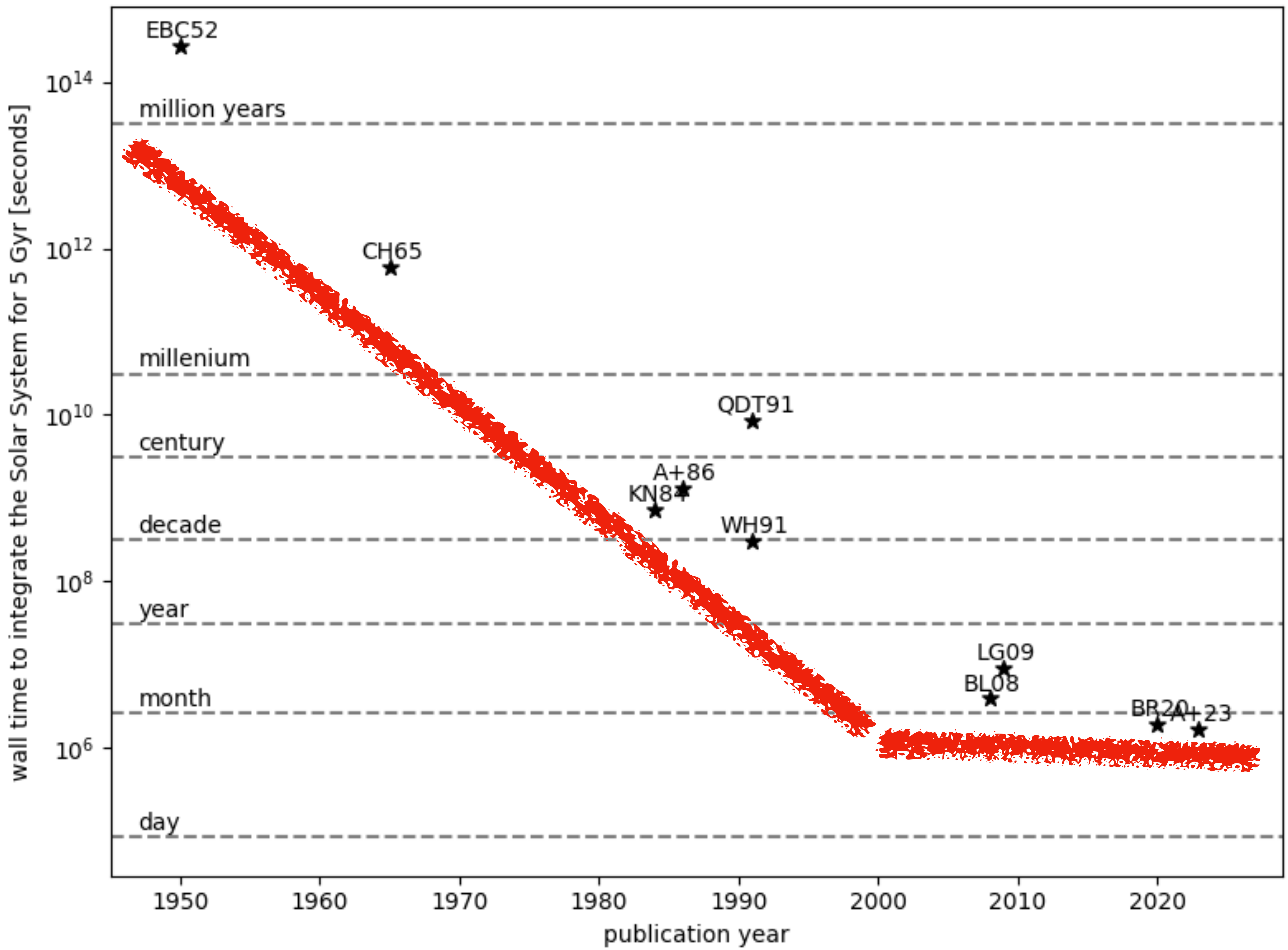
- Solving this ODE is inherently sequential
- Discretize time into timesteps
- One timestep after another
- Timestep needs to be smaller than smallest timescale in the problem
- Innermost orbital period
  - 88 days for Solar System
  - 1 day for exoplanets
- Might need 10^{13} timesteps!

N-body integrations of the Solar System



http://shadden.github.io/nbody_history/

N-body integrations of the Solar System



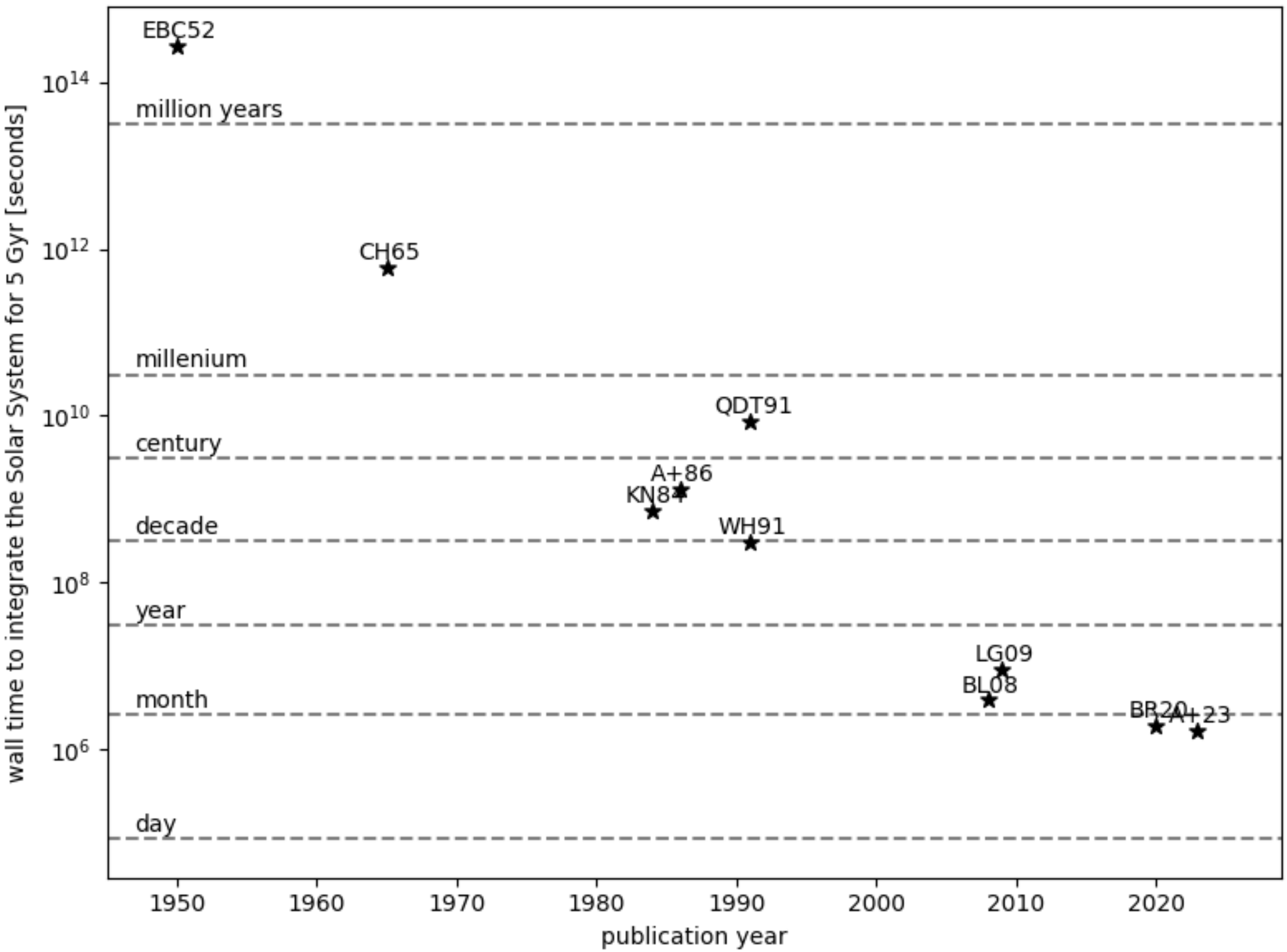
Why is it hard?

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}$$

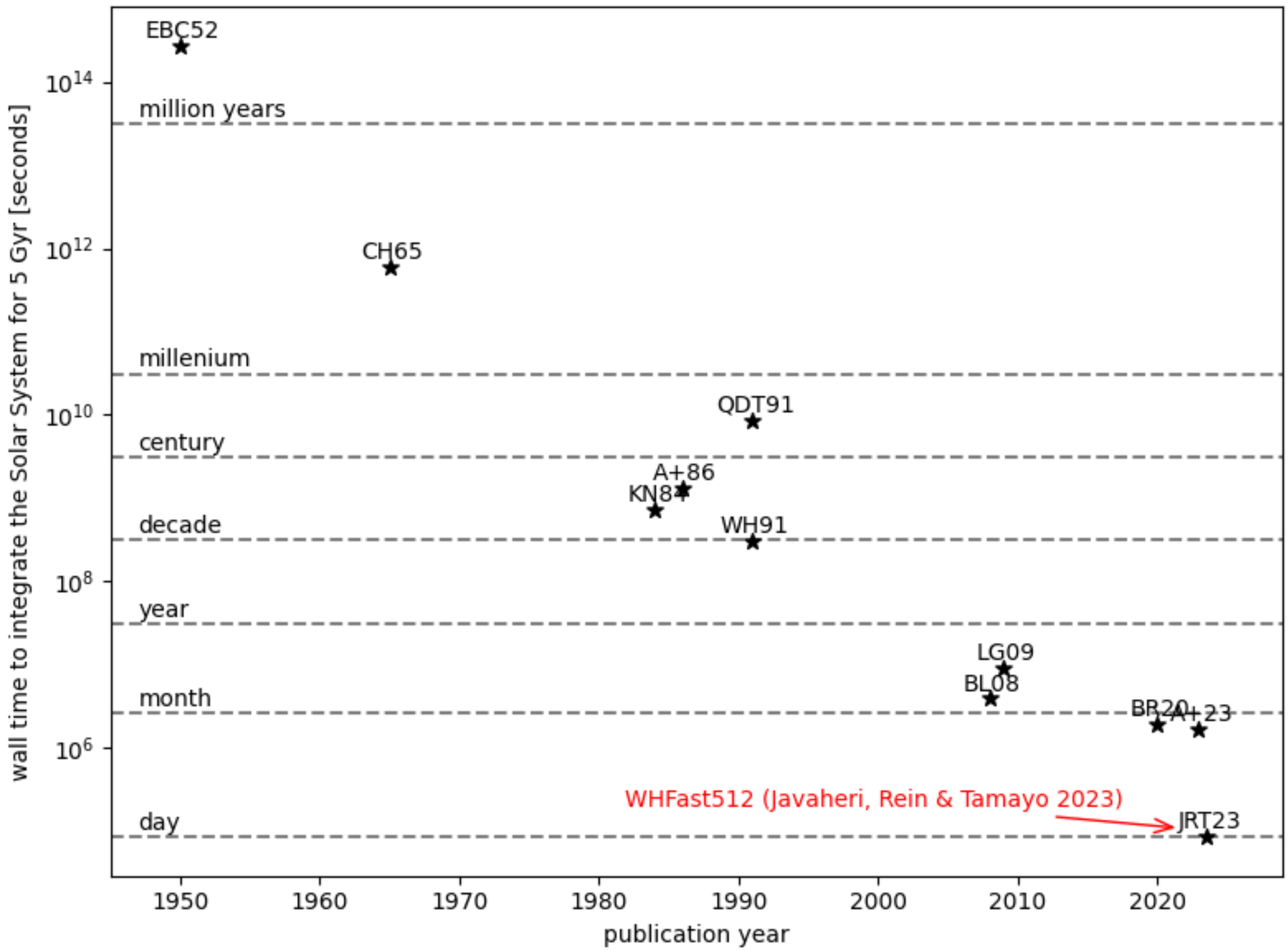
Not parallelizable*

* for small N

N-body integrations of the Solar System



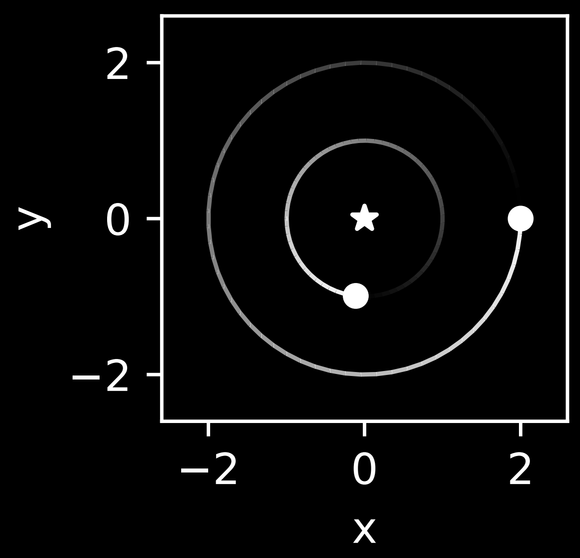
N-body integrations of the Solar System



Wisdom Holman Integrator

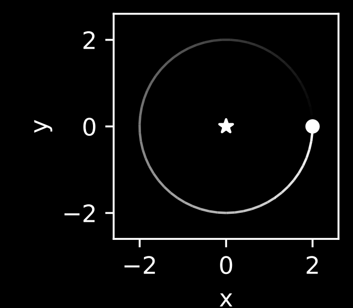
- Wisdom (1981), Wisdom & Holman (1991)
- Symplectic 2nd order integrator
- Many extensions to higher order
- Work for any gravitational system with a dominant central object

Splitting

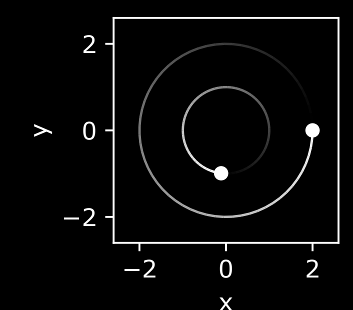


One Wisdom-Holman timestep

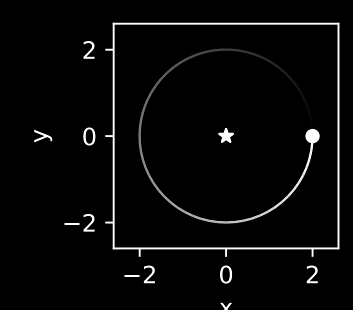
- Evolve all particles for half a timestep assuming they are on Keplerian orbits



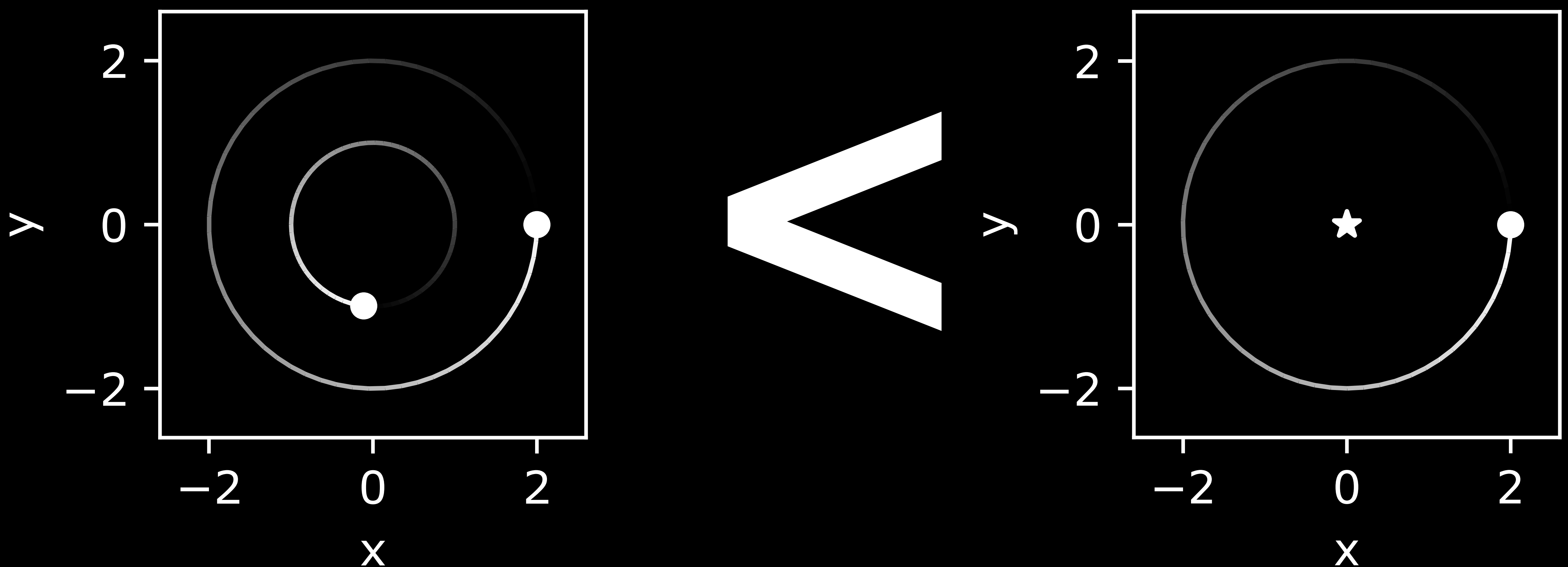
- Calculate gravitational acceleration from planet-planet interactions, update velocity assuming a full timestep



- Evolve all particles for half a timestep assuming they are on Keplerian orbits



Small perturbations



WHFAST512

AVX512

- Single Instruction Multiple Data (SIMD)
- Available on high-end Intel and AMD CPUs

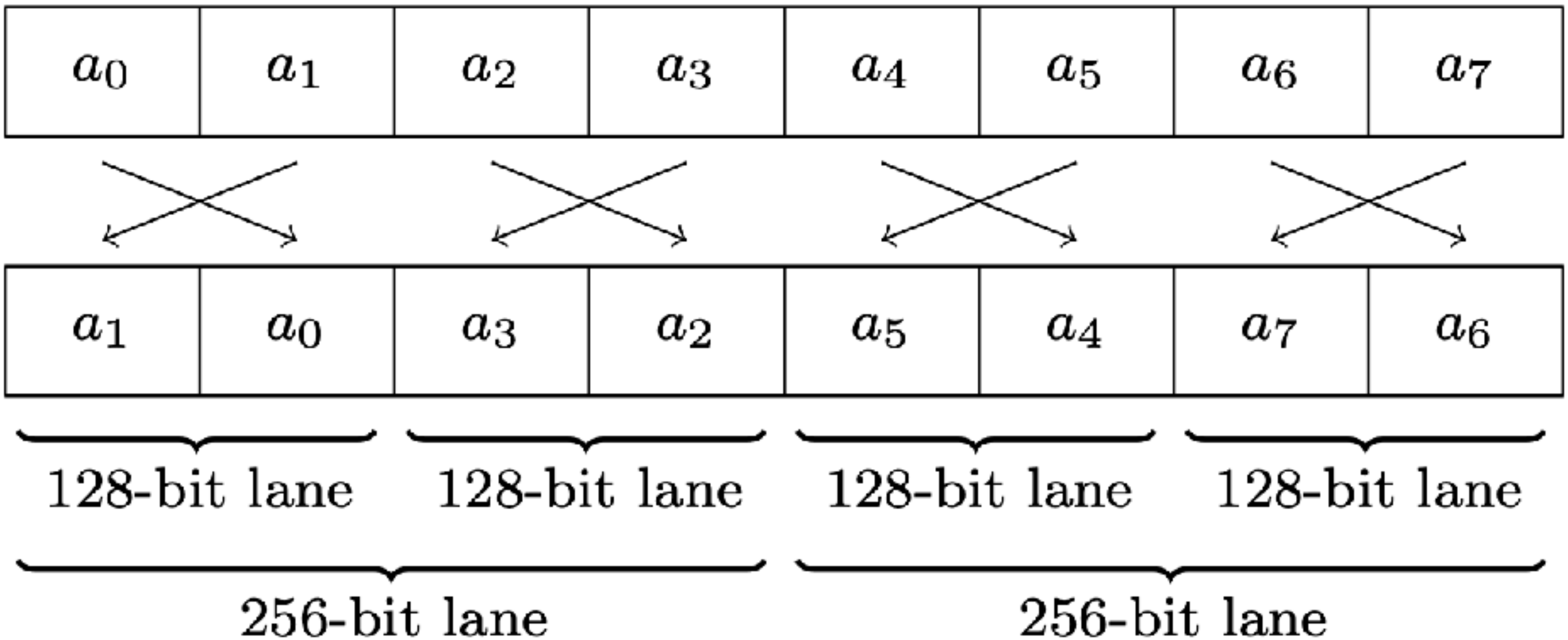
- Operates on 512 bit wide registers
- Can operate on 8 double precision floating point numbers at the same time

- Compiler can automatically produce SIMD instruction
- Or do it manually, using intrinsics / assembler

AVX512 Example: vaddpd

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
$=$	$=$	$=$	$=$	$=$	$=$	$=$	$=$
a_0+b_0	a_1+b_1	a_2+b_2	a_3+b_3	a_4+b_4	a_5+b_5	a_6+b_6	a_7+b_7

AVX512 Example: vshufpd

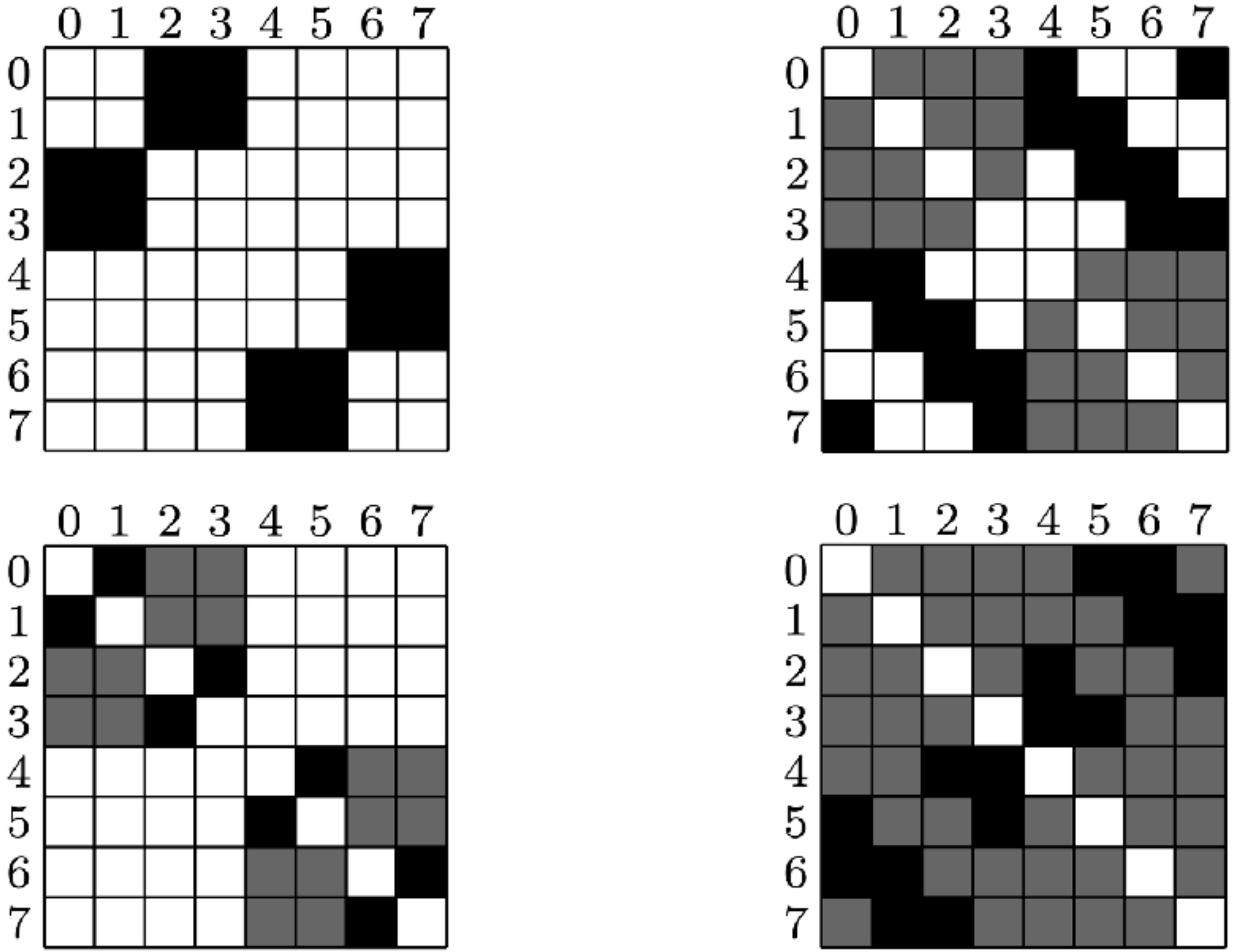


Interaction step

$$\mathbf{a}_i = -\sum_{j=1,\, j\neq i}^{N} \frac{G m_j}{Q_{ij}^3} \mathbf{Q}_{ij}$$

- Expensive parts: division and square root
- Can use symmetry of Newton's 3rd law
- How to arrange the calculations for optimal performance?

Interaction step in 4 parts



Kepler step

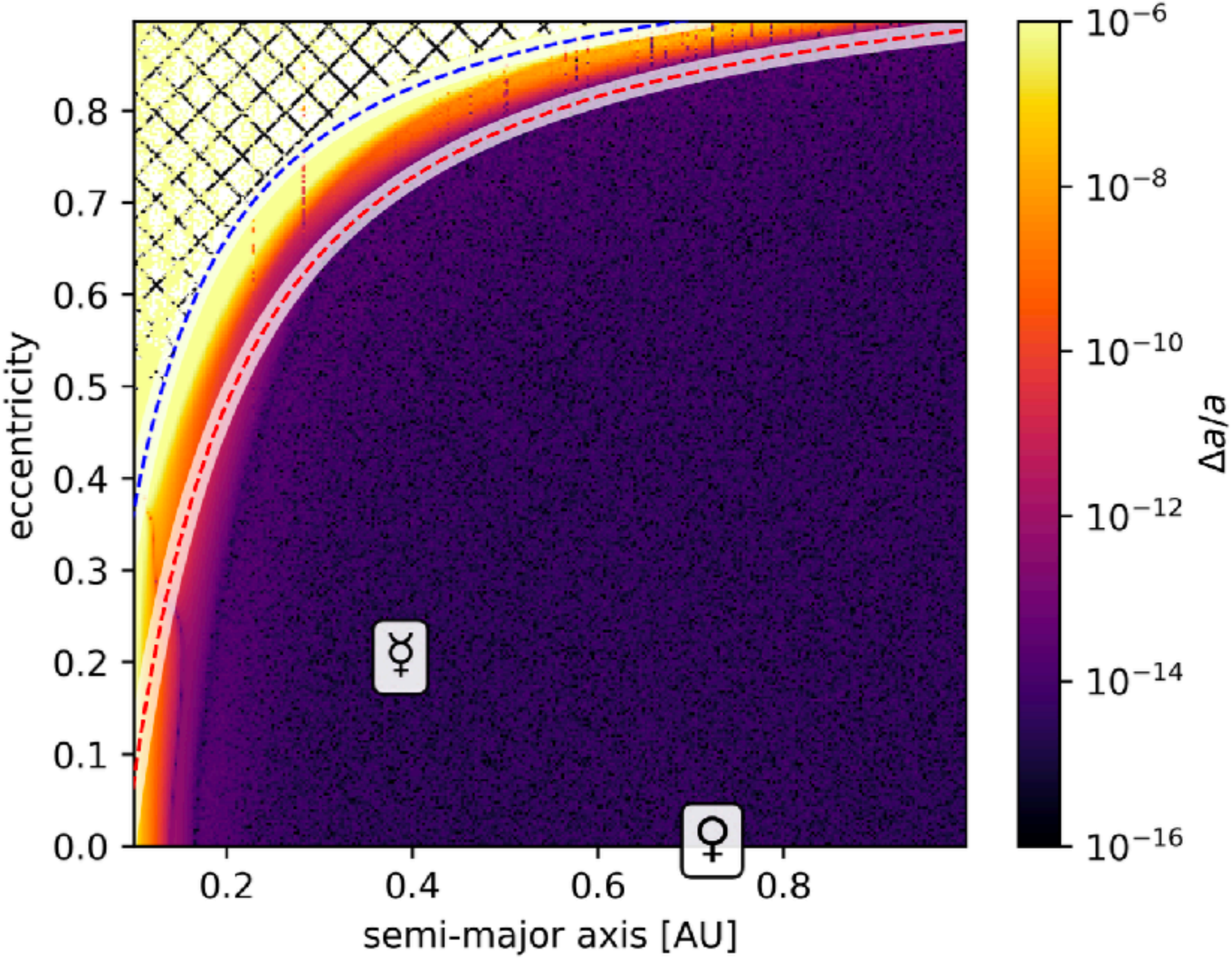
$$H_{K,i} = \frac{P_i^2}{2m_i} - \frac{Gm_0m_i}{Q_i}$$

- Iterative solution for Kepler's equation
- Fixed number of iterations using both Halley's method and Newton's method
- Instead of sin/cos, use Stumpff and Stiefel functions

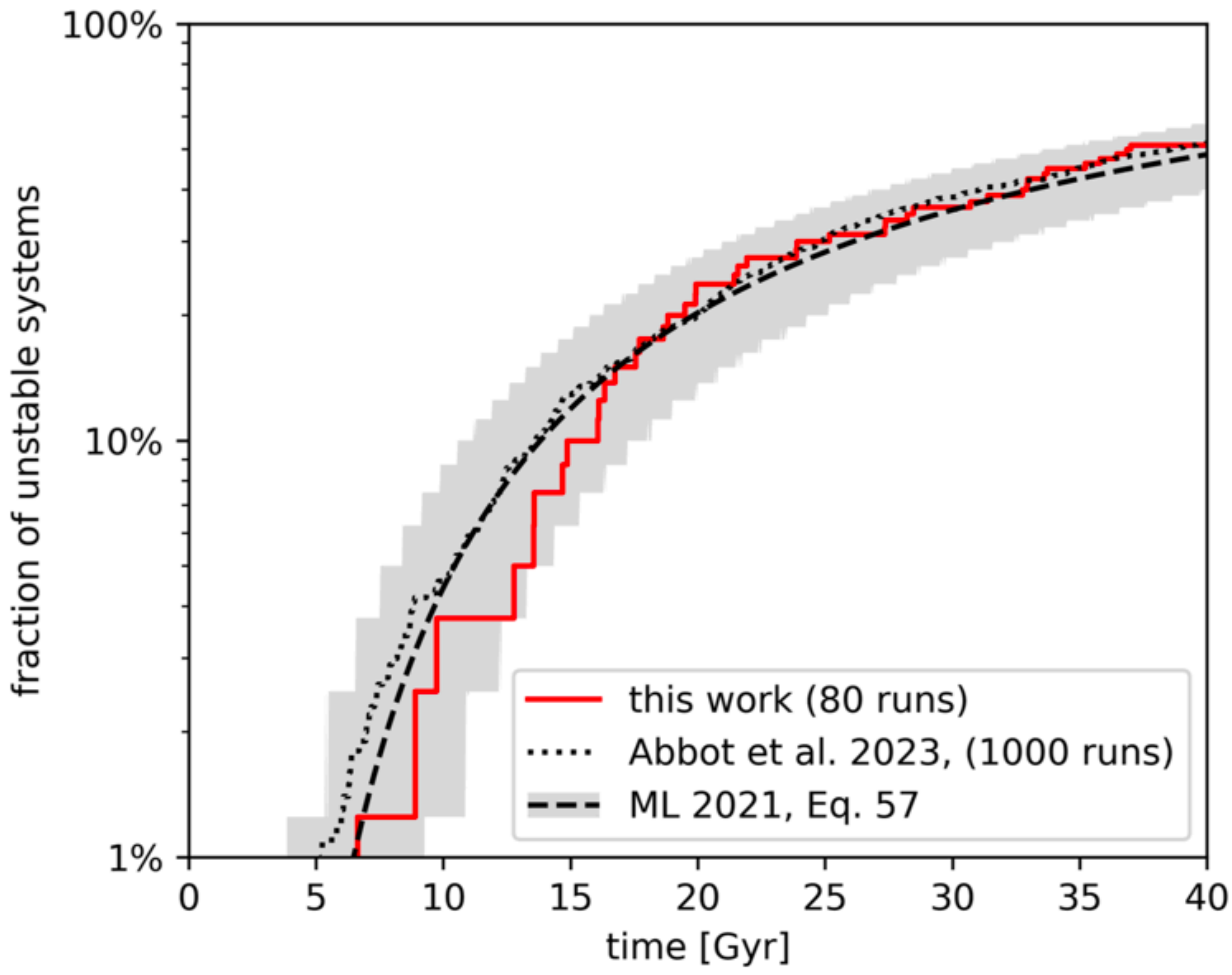
Solving Kepler's Equation in 4 iterations

Halley's method	4 terms in Stumpff function
Halley's method	4 terms in Stumpff function
Newton's method	6 terms in Stumpff function
Newton's method	6 terms in Stumpff function

Kepler solver achieves machine precision

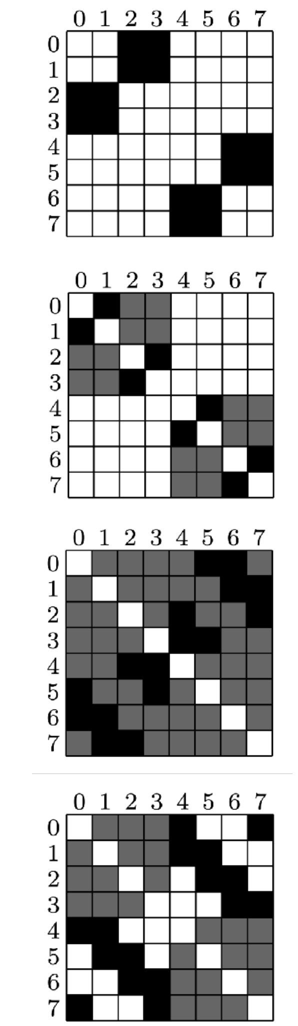


Long term integrations



Conclusions

- WHFast512 is by far the fastest N-body integrator in the world for planetary systems
- 5x - 10x faster
- Works for planetary systems with up to 8 planets
- Need a CPU which supports AVX512 instructions



Try it out! https://github.com/hannorein/rebound

Pejvak Javaheri, Hanno Rein, Dan Tamayo
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