Integrating the Solar System for 5 Gyr in just 1 day

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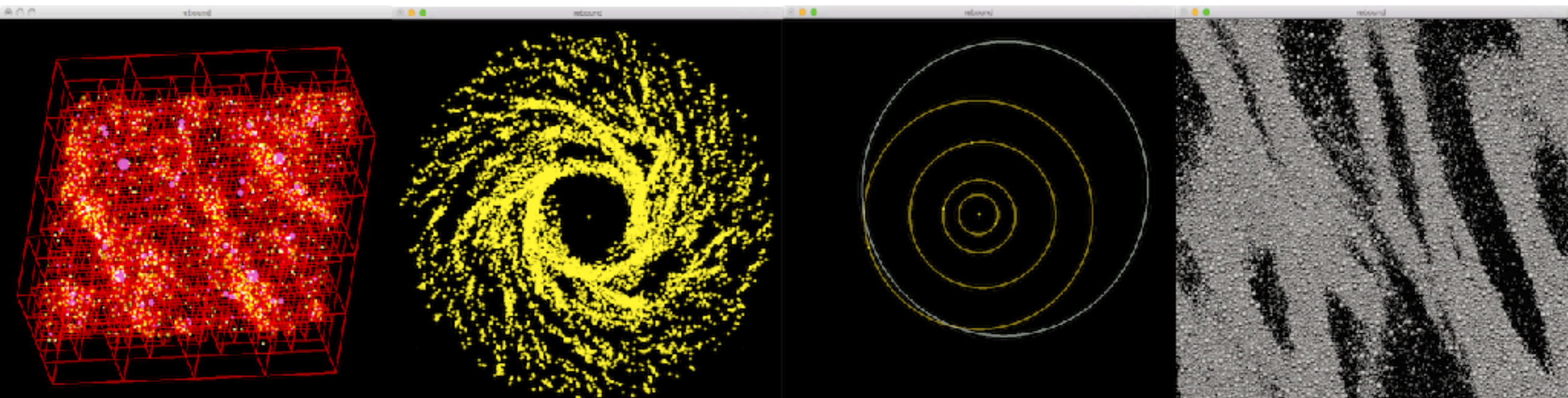
REBOUND

- N-body integrator package
- Formation / evolution of planetary systems
- Integration of Solar System objects
- Collisional simulations of planetary rings

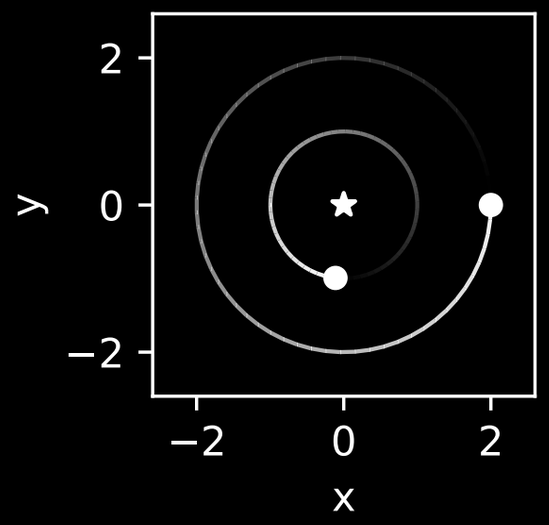
- Written in C with an easy to use python interface

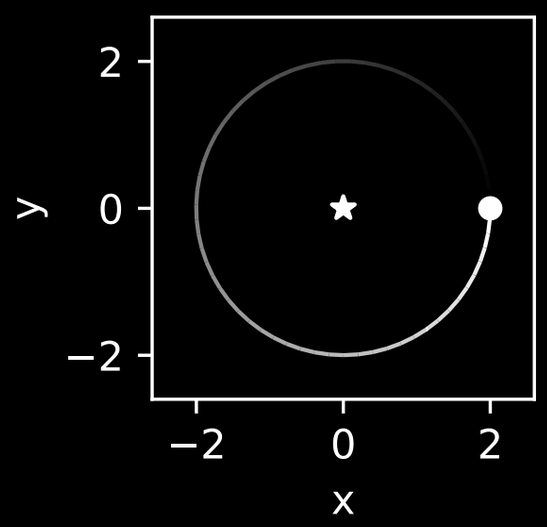


Wisdom Holman Integrator

- Wisdom (1981), Wisdom & Holman (1991)
- Symplectic 2nd order integrator
- Many extensions to higher order
- Work for any gravitational system with a dominant central object

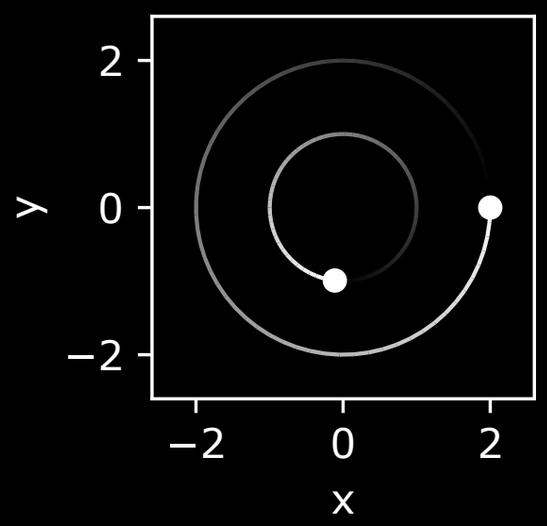
Splitting





Keplerian (Drift) Part

- Keplerian orbits
- Need to solve Kepler’s equation
- No closed form solution
- But fast iterative solution

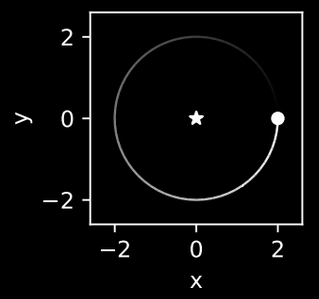


Interaction (Kick) Part

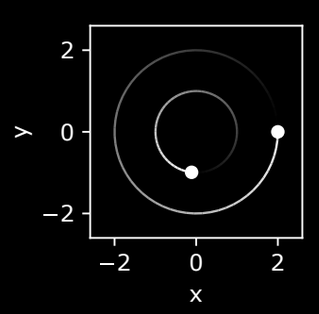
- Just involves calculating gravitational forces between planets

One Wisdom-Holman timestep

- Evolve all particles for half a timestep assuming they are on Keplerian orbits



- Calculate gravitational acceleration from planet-planet interactions, update velocity assuming a full timestep



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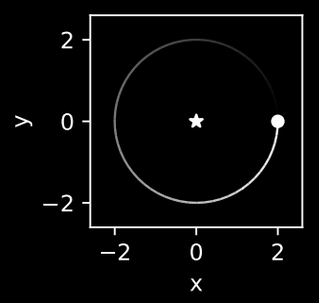
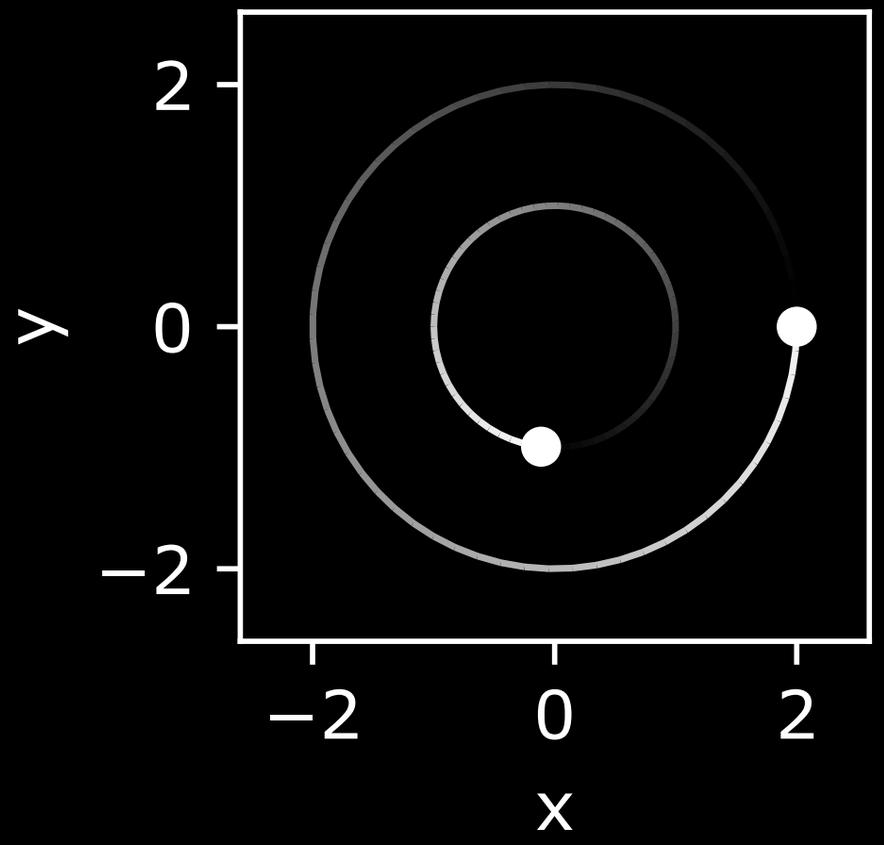


Small perturbations



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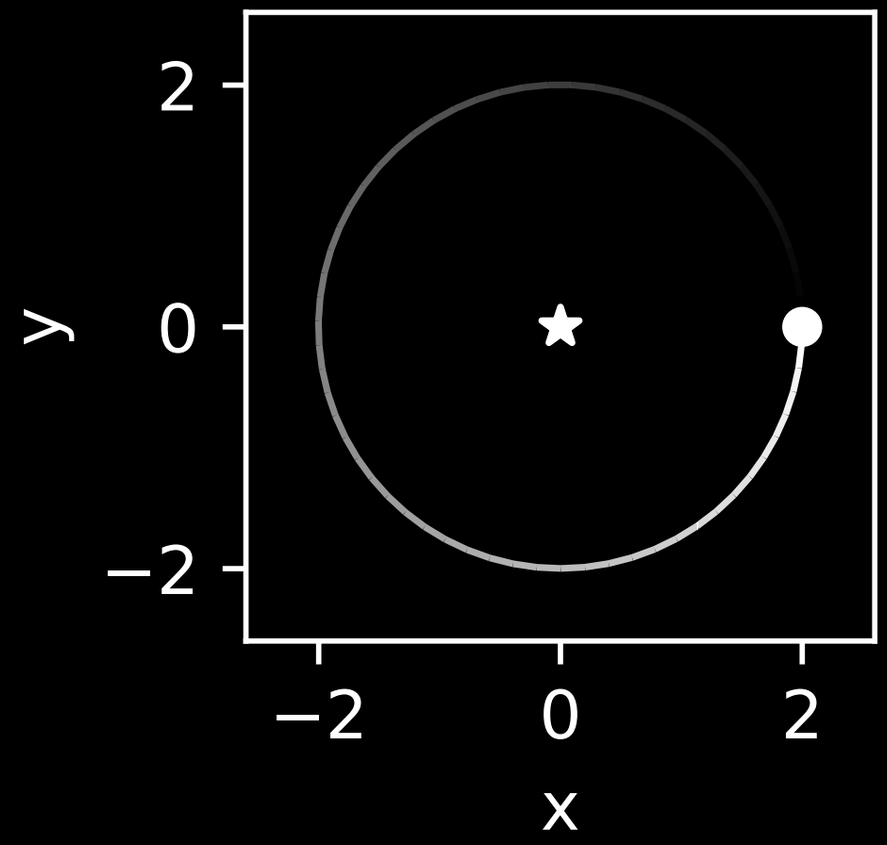

WHFAST512

CPU time to integrate the Solar System for 5 Gyr

Authors	CPU	Time
Laskar & Gastineau (2009)	Xeon 5472 CPU (3GHz)	4 months
MERCURY Chambers and Migliorini (1997)	Xeon 6148 CPU (2.4GHz)	47 days
swiftest/whm Minton et al. (2021)	Xeon 6148 CPU (2.4GHz)	13.3 days
HNBody Rauch and Hamilton (2002)	Xeon 6148 CPU (2.4GHz)	7.2 days
WHFast Rein and Tamayo (2015)	Xeon 6148 CPU (2.4GHz)	6.8 days
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WHFast512 Pejvak, Rein, Tamayo (2023)	**Xeon 6148 CPU (2.4GHz)**	**1.4 days**
WHFast512 Pejvak, Rein, Tamayo (2023)	**AMD EPYC 9654 (2.4GHz)**	**1 day**

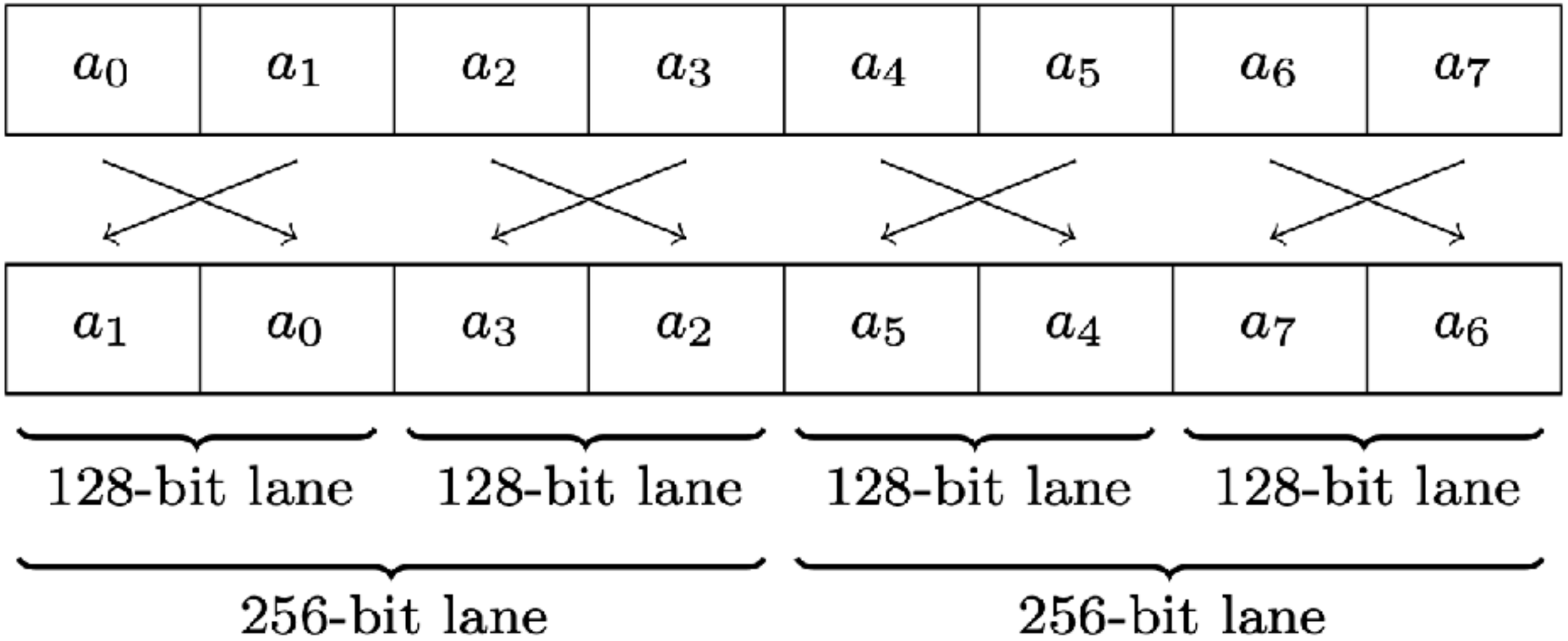
AVX512

- Single Instruction Multiple Data (SIMD)
- Available on high end Intel and AMD CPUs
- Can operate on 8 double precision floating point numbers at the same time
- Can use a compile to automatically produce SIMD instruction
- Or do it manually, using intrinsics / assembler

AVX512 Example: vaddpd

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
+	+	+	+	+	+	+	+
b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
=	=	=	=	=	=	=	=
a_0+b_0	a_1+b_1	a_2+b_2	a_3+b_3	a_4+b_4	a_5+b_5	a_6+b_6	a_7+b_7

AVX512 Example: vshufpd

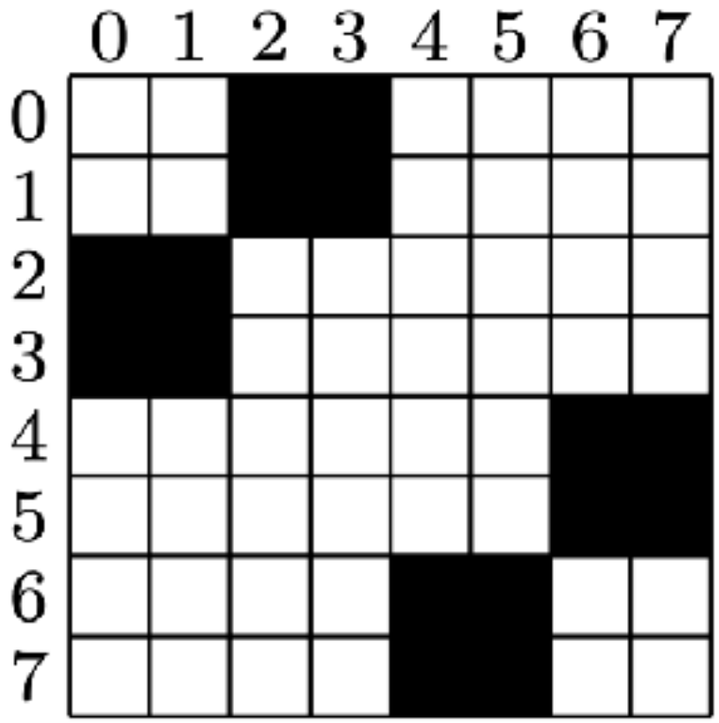


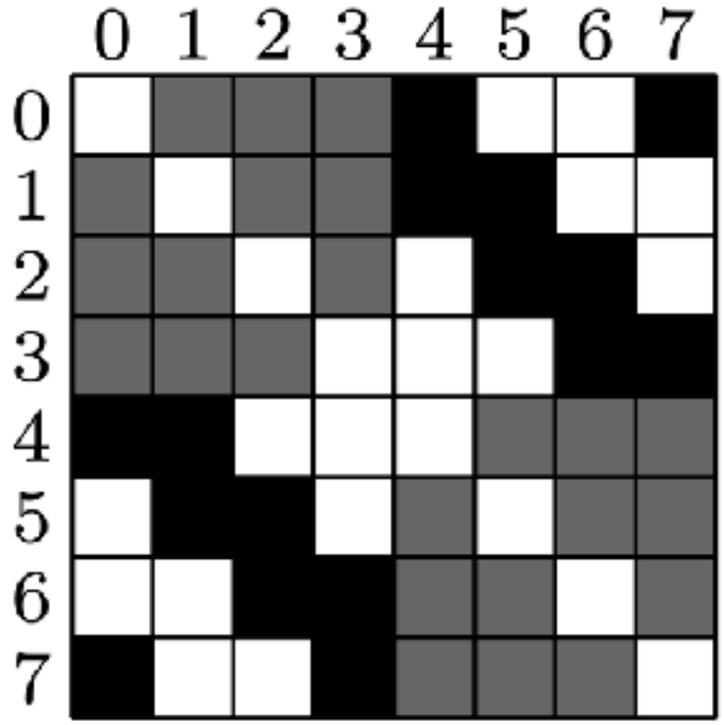
Interaction step

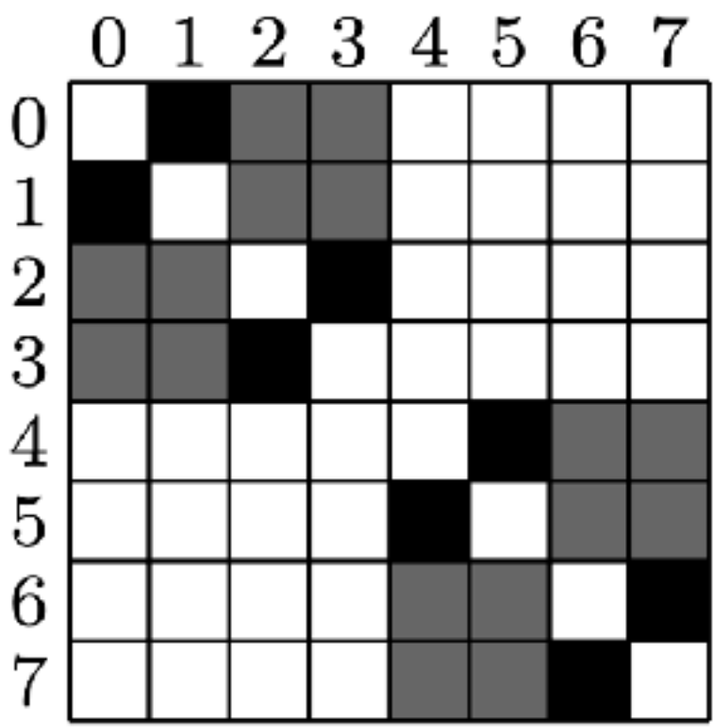
$$\mathbf{a}_i = -\sum_{j=1,\, j\neq i}^{N} \frac{Gm_j}{Q_{ij}^3}\mathbf{Q}_{ij}$$

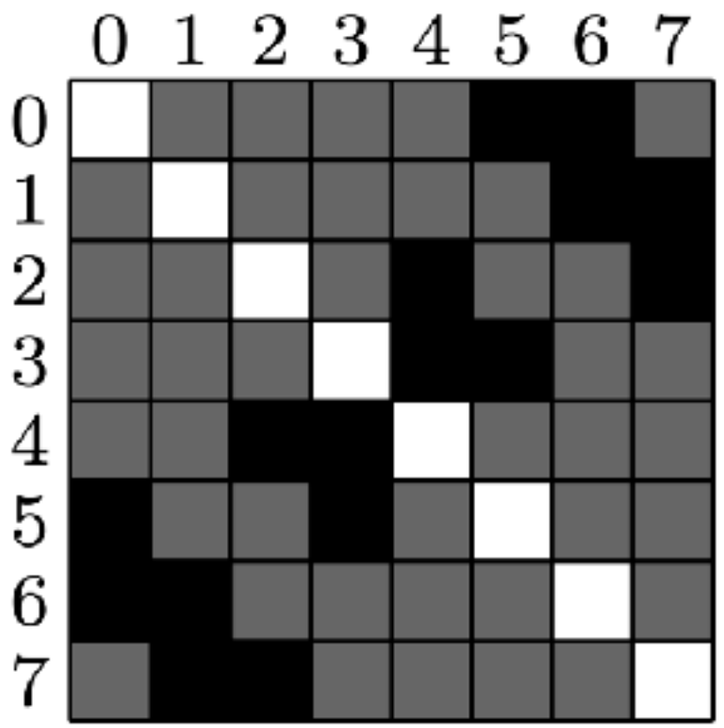
- Expensive parts: division and square root
- Can use symmetry of Newton's 3rd law
- How to arrange the calculations for optimal performance?

Interaction step in 4 parts









Kepler step

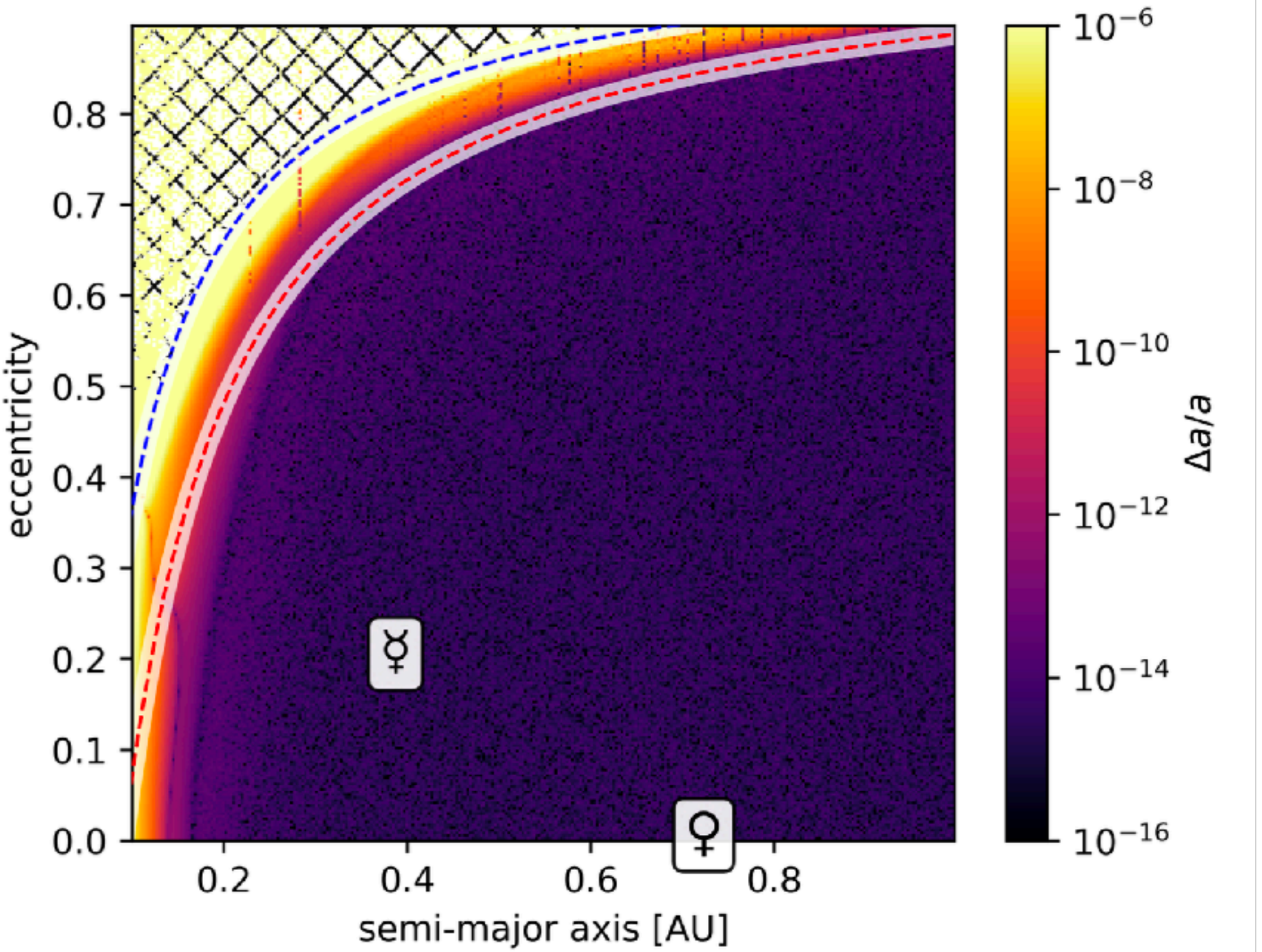
$$H_{K,i} = \frac{P_i^2}{2m_i} - \frac{Gm_0m_i}{Q_i}$$

- Iterative solution for Kepler's equation
- Fixed number of iterations using both Halley's method and Newton's method
- Instead of sin/cos, use Stumpff and Stiefel functions

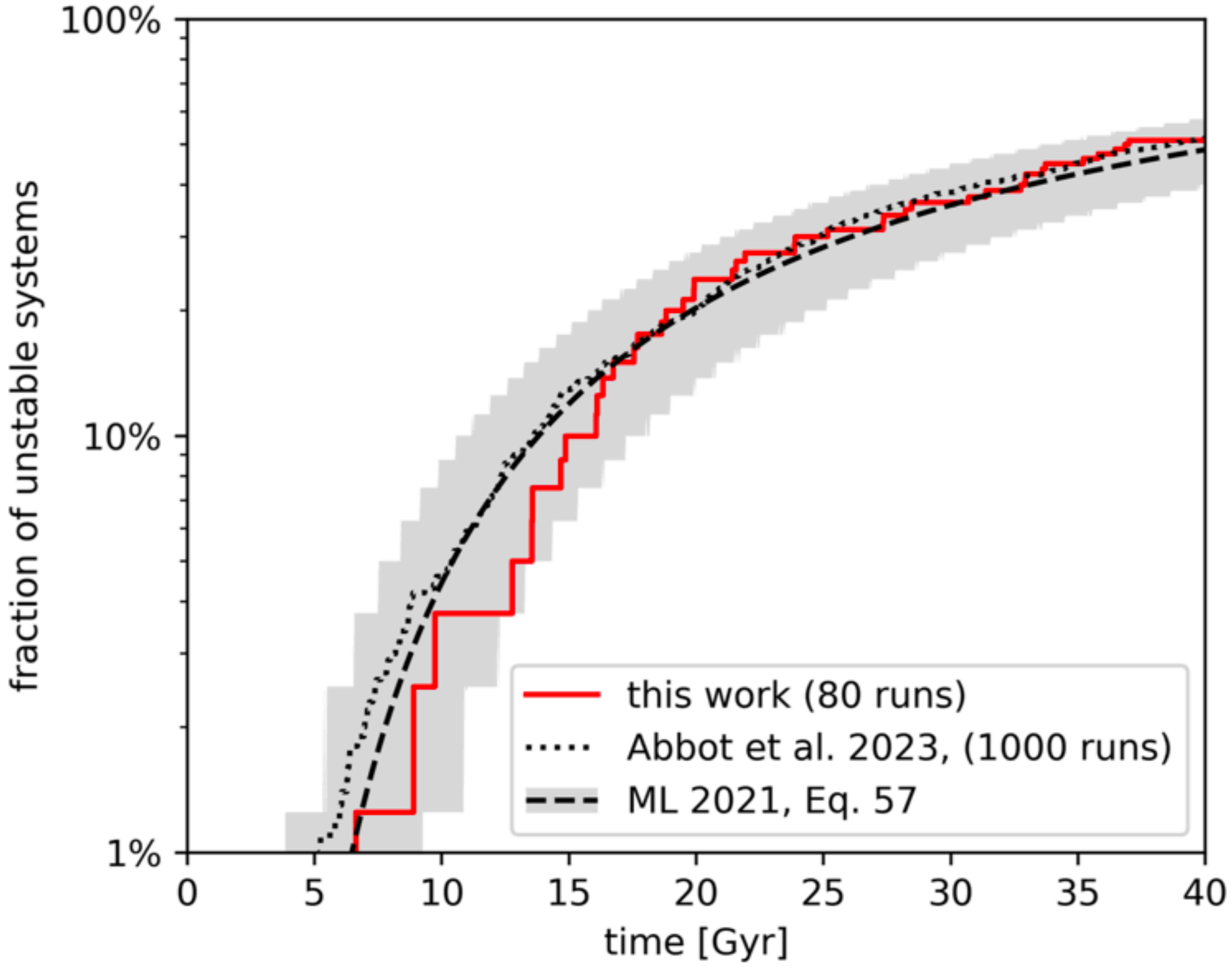
Solving Kepler's Equation in 4 iterations

Halley's method	4 terms in Stumpff function
Newton's method	6 terms in Stumpff function
Newton's method	6 terms in Stumpff function

Kepler solver achieves machine precision



Long term integrations



Conclusions

- WHFast512 is by far the fastest N-body integrator in the world
- Freely available within REBOUND package
- Need a CPU which supports AVX512 instructions

Next steps:

- Optimize WHFast512 for 2 and 4 planet systems
- Keep up-to-date with new CPUs